Since $x = s$, $y = t$ in this parametrization of the graph, we conclude that

Surface area of the graph of $f(x, y)$ over D

$$= \iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dx\, dy. \tag{9}$$

◆

One final note: It is not at all clear that either formula (6) or formula (8) depends only on the underlying surface $S = \mathbf{X}(D)$ and not on the particular parametrization $\mathbf{X}$. These formulas are independent of the parametrization, as we shall observe in the following section, in the context of general surface integrals.

1 Exercises

1. Let $\mathbf{X}\colon \mathbf{R}^2 \to \mathbf{R}^3$ be the parametrized surface given by

$$\mathbf{X}(s, t) = (s^2 - t^2, s + t, s^2 + 3t).$$

(a) Determine a normal vector to this surface at the point

$$(3, 1, 1) = \mathbf{X}(2, -1).$$

(b) Find an equation for the plane tangent to this surface at the point $(3, 1, 1)$.

2. Find an equation for the plane tangent to the torus

$$\mathbf{X}(s, t) = ((5 + 2\cos t)\cos s, (5 + 2\cos t)\sin s, 2\sin t)$$

at the point $\left((5 - \sqrt{3})/\sqrt{2}, (5 - \sqrt{3})/\sqrt{2}, 1\right)$.