4 Exercises

Calculate the divergence of the vector fields given in Exercises 1–6.

1. $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j}$

2. $\mathbf{F} = y^2\mathbf{i} + x^2\mathbf{j}$

3. $\mathbf{F} = (x + y)\mathbf{i} + (y + z)\mathbf{j} + (x + z)\mathbf{k}$

4. $\mathbf{F} = z\cos(e^{y^2})\,\mathbf{i} + x\sqrt{z^2+1}\,\mathbf{j} + e^{2y}\sin 3x\,\mathbf{k}$

5. $\mathbf{F} = x_1^2\mathbf{e}_1 + 2x_2^2\mathbf{e}_2 + \cdots + nx_n^2\mathbf{e}_n$

6. $\mathbf{F} = x_1\mathbf{e}_1 + 2x_1\mathbf{e}_2 + \cdots + nx_1\mathbf{e}_n$

Find the curl of the vector fields given in Exercises 7–11.

7. $\mathbf{F} = x^2\,\mathbf{i} - xe^y\,\mathbf{j} + 2xyz\,\mathbf{k}$

8. $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

9. $\mathbf{F} = (x + yz)\mathbf{i} + (y + xz)\mathbf{j} + (z + xy)\mathbf{k}$

10. $\mathbf{F} = (\cos yz - x)\mathbf{i} + (\cos xz - y)\mathbf{j} + (\cos xy - z)\mathbf{k}$

11. $\mathbf{F} = y^2z\,\mathbf{i} + e^{xyz}\mathbf{j} + x^2y\,\mathbf{k}$

12. (a) Consider again the vector field in Exercise 8 and its curl. Sketch the vector field and use your picture to explain geometrically why the curl is as you calculated.

(b) Use geometry to determine $\nabla \times \mathbf{F}$, where $\mathbf{F} = \dfrac{(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})}{\sqrt{x^2 + y^2 + z^2}}$.

(c) For $\mathbf{F}$ as in part (b), verify your intuition by explicitly computing $\nabla \times \mathbf{F}$.

13. Can you tell in what portions of $\mathbf{R}^2$, the vector fields shown in Figures 43–46 have positive divergence? Negative divergence?

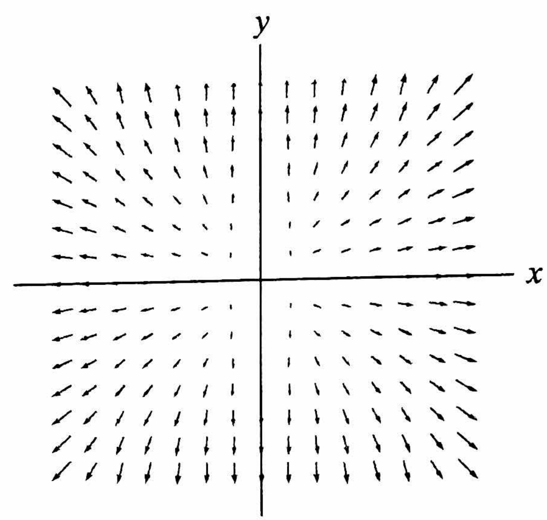

Figure 43 Vector field for Exercise 13(a).

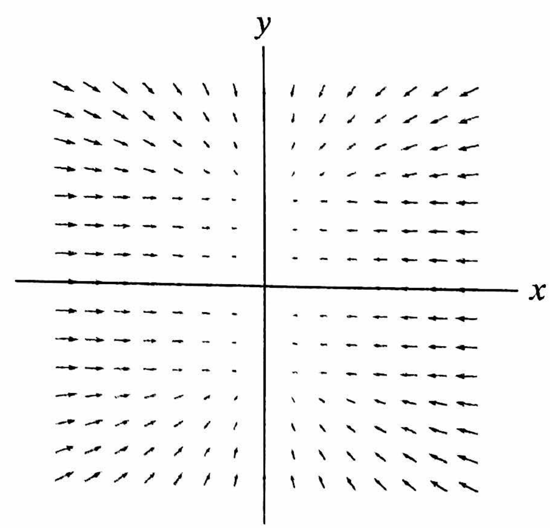

Figure 44 Vector field for Exercise 13(b).