Chapter 3 Homework Solutions

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1. (a) You are counting the number of games and there are a limited number of games in a tennis match.
   Answer: Finite discrete

   (b) your counting the nubmer of tickets.
   Answer: Infinite discrete

   (c) Time is an interval and it doesn't skip values.
   Answer: Continuous

   (d) The number may be very large(hopefully), but it is still only a fixed number.
   Answer: Finite discrete

   (e) Temperature is an interval and it doesn't skip values.
   Answer: Continuous

2. (a) There are $52 - 13 = 39$ non-heart cards in a deck, so the maximum number of cards you could draw is 39 without drawing a heart. So the worst case scenario is 40 cards drawn.

   Answer: Finite discrete.
   Values: X = 1, 2, ..., 40

   (b) Continuous
   Values: $\{x = \text{time in hours } | 0 \leq X \leq 24\}$

   (c) You could always roll a one, so it might not happen that you roll a six.
   Answer: Infinite discrete
   Values: $X = 1, 2, 3, 4, ...$

3. The areas of the rectangles must add to one since the rectangles represent probability. The missing rectangle has an area of 0.15.

   Answer: $0.15 + 0.2 + 0.3 = 0.65$ or $1 - 0.1 - 0.25 = 0.065$

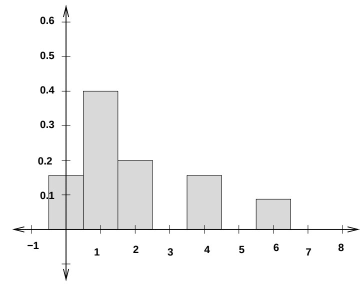
4. Let $P(X = 6) = J$ then $P(X = 3) = 2J$
   $0.1 + 0.25 + P(X = 3) + 0.2 + 0.15 + P(X + 6) = 1$ (from the histogram).
   $P(X = 3) + P(X + 6) = 0.3$
   $2J + J = 0.3$
   and get $J = 0.1$

   Answer: $0.45 = P(X = 4) + P(X = 5) + P(X = 6)$

5. (a) Divide the frequency by the total number of students who have waited to get relative frequency(or probability).

students	0	1	2	4	6
prob.	$\frac{4}{25}$	$\frac{10}{25}$	$\frac{5}{25}$	$\frac{4}{25}$	$\frac{2}{25}$

   (b) probability histogram



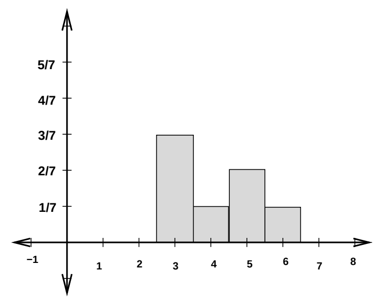
6. There are a total of 7 cards that will be made. Three of them will have a word with three letters: Get, Its, fun.

   Answer:

   (a)

letters	3	4	5	6
prob.	$\frac{3}{7}$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{1}{7}$

   (b) probability histogram



7. (a) There can be different answers depending where your intervals start.

speed(x)	freq
$25 \leq x < 30$	6
$30 \leq x < 35$	7
$35 \leq x < 40$	9
$40 \leq x < 45$	8
$45 \leq x < 50$	5
$50 \leq x < 55$	5

   (b) prob dist.

speed(x)	prob
$25 \leq x < 30$	6/40
$30 \leq x < 35$	7/40
$35 \leq x < 40$	9/40
$40 \leq x < 45$	8/40
$45 \leq x < 50$	5/40
$50 \leq x < 55$	5/40

8. (a) frequency table

grade(x)	freq
$90 \le x \le 99$	10
$80 \le x \le 89$	11
$70 \le x \le 79$	11
$60 \le x \le 69$	10
$50 \le x \le 59$	7
$40 \le x \le 49$	4
$30 \le x \le 39$	3

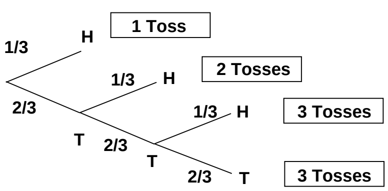
(b) prob dist.

grade(x)	freq
$90 \le x \le 99$	10/56
$80 \le x \le 89$	11/56
$70 \le x \le 79$	11/56
$60 \le x \le 69$	10/56
$50 \le x \le 59$	7/56
$40 \le x \le 49$	4/56
$30 \le x \le 39$	3/56

9. Remember that the remainder is what is left over after performing long division(by hand). For example: 7 divide by 3 has a remainder of 1 since 3 goies into 7 two times(this gives $3*2=6$) and 1 will be left over.

remainder	0	1	2
prob.	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{3}{8}$

10. The tree shows the experiment. Notice the tree stops on the third level since either a head is tossed or the coin has been tossed three times.



Use the branches to get the probability.

Answer:

tosses	1	2	3
prob.	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{4}{9}$

11. (a) $P(X=0) = \frac{C(4,0)C(48,3)}{C(52,3)}$

(b) $P(X=2) = \frac{C(4,2)C(48,1)}{C(52,3)}$

12. (a) $P(X=2) = \frac{C(5,2)*C(7,1)}{C(12,3)} = \frac{70}{220}$

(b) $P(X \le 2) =$

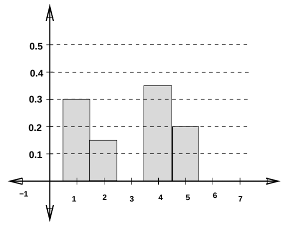
$\frac{C(5,0)*C(7,3)}{C(12,3)} + \frac{C(5,1)*C(7,2)}{C(12,3)} + \frac{C(5,2)*C(7,1)}{C(12,3)} = \frac{210}{220}$

or

$P(X \le 2) = 1 - P(X=3) = 1 - \frac{C(5,3)*C(7,0)}{C(12,3)}$

13. (a) $E(x) = 1*0.3 + 2*0.15 + 4*0.35 + 5*0.2 = 3$

(b) histogram



14. To calculate $P(X=70)$ remember that the probabilities must add to 1.
$E(X) = 30*0.31 + 32*0.25 + 46*0.29 + 49*0.06 + 63*0.04 + 70*0.05 = 39.6$

15. (a) Write out the cards and give the score to each card. Note: the order of the numbers is not important.

Card	Score	Card	Score	Card	Score
1,2	1	1,3	1	1,4	10
1,5	1	2,3	10	2,4	2
2,5	2	3,4	3	3,5	3
4,5	4				

Answer:

score	1	2	3	4	10
probability	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{2}{10}$

(b) $E(x) = 1*\frac{3}{10} + 2*\frac{2}{10} + 3*\frac{2}{10} + 4*\frac{1}{10} + 10*\frac{2}{10} = 3.7$

16. The probabilities may be computed using a tree or combinations.

(a)

hearts	0	1	2
probability	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{2}{34}$

(b) $E(x) = 0*\frac{19}{34} + 1*\frac{13}{34} + 2*\frac{2}{34} = 0.5$

17. Use a dice chart to find the probabilities.

Red Die

Green Die	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

(a)

hearts	1	2	3	4	5	6
probability	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

(b) $E(x) = 1*\frac{1}{36} + 2*\frac{3}{36} + 3*\frac{5}{36} + 4*\frac{7}{36} + 5*\frac{9}{36} + 6*\frac{11}{36}$
$E(X) = 4.47222$

18. Note: X is the **net winnings**.

(a)

X	1999	499	99	24	-1
probability	$\frac{1}{500}$	$\frac{1}{500}$	$\frac{3}{500}$	$\frac{10}{500}$	$\frac{485}{500}$

(b) $E(x) = \frac{1}{500} * 1999 + \frac{1}{500} * 499 + \frac{3}{500} * 99 + \frac{10}{500} * 24 + \frac{485}{500} * (-1) = 5.1$

19. X = profit on a chip.

X	18	-23
prob.	0.95	0.05

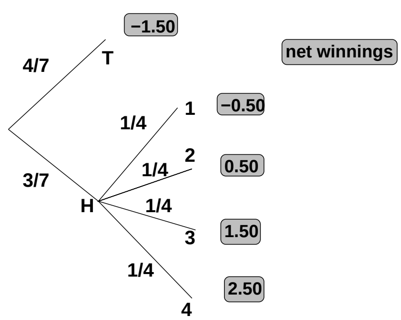
Answer: $E(x) = 18 * 0.95 + (-23) * 0.05 = 15.95$

20. X is your net winnings.

hearts	-5	-4	-1	4
probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$E(X) = (-5) * 1/8 + (-4) * 3/8 + (-1) * 3/8 + 4 * 1/8$
$E(X) = -2$

21. Use a tree to set up the probability distribution.



(a)

X	-1.5	-.5	.5	1.5	2.5
prob	$\frac{4}{7}$	$\frac{3}{28}$	$\frac{3}{28}$	$\frac{3}{28}$	$\frac{3}{28}$

(b) $E(x) = -.43$ so the game is not fair.

22. Use a tree or combinations to find the probabilities.
X is your net winnings and A be the cost of the game.

	1 red	2 red	0 red
X	4-A	3A-A	0-A
prob	$\frac{20}{36}$	$\frac{6}{36}$	$\frac{10}{36}$

If the game is fair then $E(x) = 0$

$0 = \frac{20}{36} * (4 - A) + \frac{6}{36} * (2A) + \frac{10}{36} * (-A)$
$0 = 20(4 - A) + 12A - 10A$
$18A = 80$
$A = \frac{80}{18} = 4.44$

So to make it fair(or as fair as possible) charge \$4.44.

23. X is the your net winnings.

X	2	1	-3
prob.	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{4}{6}$

(a) -1.5

(b) No, the expected winnings are negative. For this problem the game favors the person running the game.

(c) Let A =Price of the game, then solve the following equaiton,

X	$7 - A$	$6 - A$	$2 - A$
prob.	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{4}{6}$

$0 = (7 - A) * 1/6 + (6 - A) * 1/6 + (2 - A) * 4/6$
$0 = (7 - A) + (6 - A) + (2 - AA) * 4$
$A = 3.5$

Answer: \$3.50

24. Note: expected value is an average so do not round the answer.

(a) $E(X) = n * p = 80 * 0.18 = 14.4$

(b) $E(X) = n * p = 80 * 0.82 = 65.6$

25. expected number of questions correct: $10 * \frac{1}{6} = 1.66666667$

Expected grade is $10 * 1.6667 = 16.6667$

26. $E(x) = 75 * 0.05 = 3.75$ Note: expected value is an average so do not round the answer.

27. $E(x) = 6 * \frac{20}{52} = 2.30769$

28. Mean = 4.9
Median = 5
Mode = 6

29. Mean = 21.31818
Median = 20.5
Mode = 19 and 24

30. The fifth score is less than or equal to 82 since 82 is the median and there are 2 scores that are above this number.

31. Answers will vary. I used the midpoint of each interval
$\frac{2.5*8+8.5*12+15*24+22*35}{8+12+24+35} = 15.8481$

32. Answers will vary. used the median of each interval.

Estimated Mean: 30.96

33. Enter the x-value in list 1 and the frequency in list 2. use the command: 1-Var Stats L_1,L_2

(a) mean: $\overline{x} = 3.75$
median = 4
mode = 4
standard deviation: $\sigma_x = 1.25$
variance: $(\sigma_x)^2 = 1.5625$

(b) mean: $\overline{x} = 7.3333$
median = 4
mode = 1 and 15
standard deviation: $\sigma_x = 6.315765$
variance: $(\sigma_x)^2 = 39.88888754$

34. Enter the x-value in list 1 and the frequency in list 2. use the command: 1-Var Stats L_1,L_2

(a) mean: $\overline{x} = 41.8023$

(b) median = 31.5

(c) mode = 90

(d) standard deviation: $S_x = 32.8697$

(e) variance: $(S_x)^2 = 1080.4171$

(f) $Q_1 = 12$ At least 25% of the people surveyed drink 12 or fewer Dr. Peppers during the semester.
Q_2 = median = 31.5 At least 50% of the people surveyed drink 31.5 or fewer Dr. Peppers during the semester.
$Q_3 = 90$ At least 75% of the people surveyed drink 90 or fewer Dr. Peppers during the semester.

35. Answers will vary. I used the middle of each interval.

(a) mean = 11.42333

(b) standard deviation: $\sigma_x = 6.561437$

(c) 11-20

36. Enter the age in list 1 and the frequency in list 2. use the command: 1-Var Stats L_1,L_2

(a) Mean = 2.6225
Median = 3
Mode = 3

(b) $Q_1 = 2$ At least 25% of the cars are 2 years or younger.
Q_2 =median = 3 At least 50% of the cars are 3 years or younger.
$Q_3 = 3$ At least 75% of the cars are 3 years or younger.

(c) Sample since there are more than 2000 cars on campus.

(d) $S_x = 1.623672352$

(e) mean $+S_x = 4.2462$
mean $-S_x = 0.9988$
Between 0.9988 years and 4.2462 years

(f) mean $+1.6 * S_x = 5.2204$
mean $-1.6 * S_x = 0.0246$
Between 0.0246 years and 5.2204 years

37. Create a probability distribution from the histogram. Enter the x-values in list 1 and the probability in list 2. use the command: 1-Var Stats L_1,L_2

(a) $E(x) = \overline{x} = 3.5$

(b) $\sigma_x = 1.62788206$

(c) varience = $(\sigma_x)^2 = 2.650000001$

38. E(X)= $n * p = 8 * \frac{1}{5} = 1.6$
st. dev. = $\sqrt{n * p * q} = \sqrt{8 * \frac{1}{5} * \frac{4}{5}} = 1.13137$

39. $\sqrt{20 * \frac{2}{20} * \frac{18}{20}} = 1.9365$

40. (a) $\mu = 80 * .15 = 12$
$\sigma = \sqrt{80 * .15 * .85} = 3.1937$

(b) within 1 standard deviation means

$\mu - 1 * \sigma \leq X \leq \mu + 1 * \sigma$
$8.806 \leq X \leq 15.19$ or
$x = 9, 10, 11, 12, 13, 14, 15$
binomcdf(80,0.15,15) − binomcdf(80,0.15,8)
Answer: 0.7283

(c) X = 7, 8, 9, ...,17

binomcdf(80,0.15,17) − binomcdf(80,0.15,6)
Answer: 0.9175

41. Use Chebychev's inequality.
$\mu + k\sigma = 27.2$
$20 + k * 2.4 = 27.2$
$k = 3$
$P(12.8 \leq X \leq 27.2) \geq 1 - \frac{1}{3^2} = \frac{8}{9}$

42. Use Chebychev's inequality.
$\mu + k\sigma = 37.3$
$35 + k * 4.5 = 37.3$
$k = 0.6$
$P(32.3 \leq X \leq 37.7) \geq 1 - \frac{1}{0.6^2} = -1.77777$
Note: Chebyshev's inequality doesn't really give useful information for this problem.

43. Use Chebychev's inequality.

(a) $\mu + k\sigma = 213$
$213 = 205 + 2 * k$
$k = 4$
$P(197 \leq X \leq 213) \geq 1 - \frac{1}{4^2}$
Answer: $\geq .9375 = \frac{15}{16}$

(b) Want to compute: $P(X < 185) + P(X > 225)$
notice that:
$P(X < 185) + P(X > 225) = 1 - P(185 \leq X \leq 225)$
$\mu + k\sigma = 225$
$225 = 205 + 2k$
$k = 10$
$P(185 \leq X \leq 225) \geq 1 - \frac{1}{10^2} = 0.99$
Answer: ≤ 0.01

44. Use Chebychev's inequality.
$\mu + k\sigma = 106$

$100 + k * 2.8 = 106$
$k = \frac{15}{7}$
$P(94 \le X \le 106) \ge 1 - \frac{1}{(15/7)^2} = 0.782222$

We would expect at least $0.78222 * 10000$ or at least 7822 boxes to have between 94 and 106 paperclips.

45. (a) normalcdf(1.25,1E99,0,1) = 0.1056
(b) normalcdf(−1, 1.5, 0, 1) = 0.7745
(c) normalcdf(−0.75, 1E99, 0, 1) = 0.7734
(d) normalcdf(−1E99, 2.5 0, 1) = 0.9938
(e) 0, since z is a continuous random variable.
(f) normalcdf(−1E99,−1,0,1) +
normalcdf(1.15,1E99,0,1)
Answer: 0.2837
(g) A = invnorm(0.647,0,1) = 0.3772
(h) J=invNorm(1−.791,0,1) = −0.8099

46. area not between A and $-A$ is $1-0.76 = 0.24$
Area at each end of the graph is $\frac{0.24}{2} = 0.12$
A = invnorm(0.12+0.76,0,1) = 1.174986

47. (a) normalcdf(111,135,100,20) = 0.268478
(b) normalcdf(85,120,100,20) = 0.614717
(c) normalcdf(75,1E99,100,20) = 0.89435
(d) A = invnorm(0.42,100,20) = 95.96213

48. (a) normalcdf(144,156,140,8) = 0.285787
(b) normalcdf(130,156,140,8) = 0.8716
(c) normalcdf(−1E99,148,140,8) = 0.8413447
(d) zero since X is a ocntinuous random variable
(e) B = invnorm(1−.37,140,8) = 142.6548268

49. (a) $\mu + 1.5\sigma = 65 + 1.5 * 6 = 74$
$\mu - 1.5\sigma = 65 - 1.5 * 6 = 56$
normalcdf(56, 74, 65,6) = 0.8663855
Answer: 86.63855%
(b) $\mu + 2\sigma = 65 + 2 * 6 = 77$
normalcdf(77, 1E99, 65,6) = 0.02275
Answer: 2.275%

50. area to the left of X=50
normalcdf(−1E99,50,50,10) = 0.5
Area to the right of B is
$1 - 0.5 - 0.48 = 0.02$
Area to the left of A is $1 - .75 - .02 = 0.23$
Answer: A =invnorm(0.23,50,10) = 42.6115

51. st. dev = $\sqrt{var} = \sqrt{225}$ =15
area to the left of X=35
normalcdf(−1E99,35,45,15) = 0.2525

Answer: A = invnorm(0.2525+0.4,45,15) = 50.8809

52. normalcdf(−1E99,112,120,10) = 0.2111855

53. (a) normalcdf(27000,1E99,24000,1400) = 0.01606
(b) normalcdf(22500,28000,24000,1400) = 0.85587
(c) binompdf(4,0.85587,2) = 0.091301

54. $\sigma = 15 * 24 = 360$
(a) normalcdf(8250, 1E99,8000,360) = 0.2437
(b) binompdf(4, 0.2437,4) = 0.003527
(c) $400 * 0.2437 = 97.48$
approximately 97

55. (a) normalcdf(28,1E99,20,5) = 0.0548
(b) since the random variable is continuous, the probability that it takes exactly 20 minutes is zero.
(c) normalcdf(16,26,20,5) = 0.6731
$500 * 0.6731 = 336.55$
approximately 336 or 337.

56. invnorm(0.8,10,2.5) = 12.10405 minutes

57. (a) normalcdf(9.2,1E99,7.4,1.2) = 0.0668
(b) 0, since this is a continous random variable

58. (a) minimum length = $1.001 - 2 * 0.002 = 0.997$
maximum length = = $1.001 + 2 * 0.002 = 1.005$
(b) normalcdf(0.997, 1.005,1.001,0.002) = 0.9545
Accept = 95.45%

Answer: 100-95.45 = 4.55%
(c) $10000 * 0.0455 = 455.$

59. (a) normalcdf(30,1E99,28.6, 2.3) = 0.2714
(b) 0, since this is a continous random variable
(c) normalcdf(28,32,28.6, 2.3) = 0.5332

60. (a) normalcdf(14,1E99, 14.1, 0.2) = 0.6915
(b) normalcdf(13.8,14.5,14.1, 0.2) = 0.9104
(c) $\mu + 1.5\sigma = 14.1 + 1.5 * 0.2 = 14.4$
$\mu - 1.5\sigma = 14 - 1.5 * 0.2 = 13.8$
normalcdf(13.8, 14.4, 14.1, 0.2) = 0.866386
Answser: 86.6386%

61. (a) normalcdf(144,1E99, 128, 14) = 0.1265
(b) noramlcdf(−1E99, 108, 128,14) = 0.07656
$250 * 0.07656 = 19.14$
Answer: about 19

62. (a) normalcdf(45, 1E99, 42, 2) = 0.0668
(b) normalcdf(−1E99, 36,42,2) = 0.0013
Answer: 0.13%

63. normalcdf(2.2, 1E99,1.5, 0.4) = 0.040059
$120 * 0.040059 = 4.807$
Answer: approximately 5

64. invnorm(0.03,20, 15/12) = 17.649 years

65. A = invnorm(1−0.08,63,15) = 84.076
B = invnorm(1 − 0.08 − 0.18, 63,15) = 72.65
C = invnorm(1 − 0.08 − 0.18 − 0.25,63,15) = 62.624