

Week in Review # 11

Section G.1: Decision Theory

- Payoff matrix
 - in terms of row player
- Pure strategy
- Saddle Point
- Strictly determined Game
- value of the game

1. John has two cards: a red 2 and a black 10. David has three cards: a red 6, a black 7 and a black 8. The game that they are going to play is to each pick a card and play it at the same time. If the cards are the same color, then John receives the difference of the two numbers. If the cards are different colors, David receives the minimum of the two numbers.

(a) Give the payoff matrix from John's point of view.

(b) Which pure strategy would be optimal for the "row" player?

(c) Which pure strategy would be optimal for the "column" player?

2. Here is the payoff matrix for a two person zero-sum game.

$$A = \begin{array}{l} \text{R-1} \\ \text{R-2} \\ \text{R-3} \end{array} \begin{bmatrix} \text{C-1} & \text{C-2} & \text{C-3} & \text{C-4} \\ 3 & -2 & -2 & 5 \\ -3 & 1 & 2 & 4 \\ 5 & -1 & 0 & -5 \end{bmatrix}$$

(a) Which pair(s) of strategies would result in the largest possible gain for the row player?

(b) Which pair(s) of strategies would result in the largest possible gain for the column player?

3. Determine if these payoff matrices represent strictly determined games? If yes, then give the optimal strategies for the row and column players and give the value of the game.

$$(a) \begin{bmatrix} 4 & -3 \\ 5 & 4 \end{bmatrix}$$

$$(b) \begin{bmatrix} 3 & -2 \\ 1 & 5 \\ -4 & 0 \\ 5 & -3 \end{bmatrix}$$

$$(c) \begin{bmatrix} 43 & 1 & -7 & -2 \\ 5 & 2 & 6 & 2 \\ -4 & 0 & -2 & 1 \\ 5 & 2 & 7 & 4 \end{bmatrix}$$

Section G.2: Mixed Strategy Games

- Expected value
- optimal strategy
- Dominance of rows/columns

4. John has two cards: a red 5 and a black 10. David has three cards: a red 6, a black 7 and a black 8. The game that they are going to play is to each pick a card and play it at the same time. If the cards are the same color, then John receives the difference of the two numbers. If the cards are different colors, David receives the minimum of the two numbers. Let A be the payoff matrix from John's point of view.

$$A = \begin{array}{c} \text{red 2} \\ \text{black 10} \end{array} \begin{bmatrix} \text{Red 6} & \text{Black 7} & \text{Black 8} \\ 4 & -2 & -2 \\ -6 & 3 & 2 \end{bmatrix}$$

- (a) David has decided to play the red 6 25% of the time and the black 8 45% of the time. Which of these strategies would be the best for John.

$$P_1 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0.35 & 0.75 \end{bmatrix}$$

- (b) What would be John's expected results if they repeat the game 1,000 times if David uses his strategy and John uses the P_3 strategy from part (a)?

5. Reduce the payoff matrix by eliminating any row or column that is dominated.

$$(a) \begin{bmatrix} 5 & -2 \\ 1 & -1 \\ -2 & 7 \end{bmatrix}$$

$$(b) \begin{bmatrix} 8 & -1 & 6 \\ -3 & 0 & 2 \end{bmatrix}$$

$$(c) \begin{bmatrix} 6 & -5 & 1 & 9 \\ 2 & 0 & 4 & 7 \\ -3 & -6 & 0 & 3 \end{bmatrix}$$

6. Find the optimal strategies for both players and the value of the game.

$$(a) \begin{bmatrix} 5 & -8 \\ 3 & 6 \end{bmatrix}$$

$$(b) \begin{bmatrix} 5 & 2 \\ 1 & -1 \\ -2 & 7 \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 2 & 6 & -4 \\ -1 & -3 & -2 & 1 \end{bmatrix}$$

$$(d) \begin{bmatrix} -1 & -3 & 1 & 0 \\ 0 & -2 & 4 & 0 \\ 3 & 1 & -3 & 2 \end{bmatrix}$$