Homogeneous operators, Sz.-Nagy – Foias Characteristic function and projective representations of $SU(1,1)$

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An operator T from a Hilbert space into itself is said to be *homogeneous* if $\varphi(T)$ is unitarily equivalent to T for all φ in Möb, the group of bi-holomorphic automorphisms of the unit disc, which are analytic on the spectrum of T. We say that a projective unitary representation σ of Möb is *associated* with an operator T if $\varphi(T) = \sigma(\varphi)^* T \sigma(\varphi)$ for all φ in Möb. A huge number of (unitarily inequivalent) examples of homogeneous operators are known. We prove that if T is a cnu contraction with associated (projective unitary) representation σ, then there is a unique projective unitary representation $\widehat{\sigma}$, extending σ, associated with the minimal unitary dilation W of T. The representation $\widehat{\sigma}$ is given in terms of σ by the formula

$$\widehat{\sigma} = (\pi \otimes D_1^+) \oplus \sigma \oplus (\pi_* \otimes D_1^-),$$

where $D_1^\pm$ are the two Discrete series representations (one holomorphic and the other anti-holomorphic) living on the Hardy space $H^2(\mathbb{T})$, and π, π_* are representations of the Möbius group living on the two defect spaces of T which are explicitly defined in terms of σ by a rather mysterious formulae. This is joint work with B. Bagchi.