Homogeneous operators, Sz.-Nagy – Foias
Characteristic function and projective representations of $SU(1, 1)$

Gadadhar Misra
Indian Institute of Science

An operator $T$ from a Hilbert space into itself is said to be homogeneous if $\varphi(T)$ is unitarily equivalent to $T$ for all $\varphi$ in Möb, the group of bi-holomorphic automorphisms of the unit disc, which are analytic on the spectrum of $T$. We say that a projective unitary representation $\sigma$ of Möb is associated with an operator $T$ if $\varphi(T) = \sigma(\varphi)^* T \sigma(\varphi)$ for all $\varphi$ in Möb. A huge number of (unitarily inequivalent) examples of homogeneous operators are known. We prove that if $T$ is a cnu contraction with associated (projective unitary) representation $\sigma$, then there is a unique projective unitary representation $\hat{\sigma}$, extending $\sigma$, associated with the minimal unitary dilation $W$ of $T$. The representation $\hat{\sigma}$ is given in terms of $\sigma$ by the formula

$$\hat{\sigma} = (\pi \otimes D_1^+) \oplus \sigma \oplus (\pi_* \otimes D_1^-),$$

where $D_1^\pm$ are the two Discrete series representations (one holomorphic and the other anti-holomorphic) living on the Hardy space $H^2(T)$, and $\pi, \pi_*$ are representations of the Möbius group living on the two defect spaces of $T$ which are explicitly defined in terms of $\sigma$ by a rather mysterious formulae. This is joint work with B. Bagchi.