1. Find the matrix of the following system and write the system in the vector form:

\[
\begin{align*}
3x_1 - 2x_2 &= 5 \\
2x_2 + x_1 &= 7
\end{align*}
\]

2. Compute \(Ax\), where \(A = \begin{pmatrix} 2 & -1 \\ \pi & 0 \end{pmatrix}\), \(x = \begin{pmatrix} 3 \\ 1 \end{pmatrix}\).

3. Compute \(AB\) and \(BA\), where \(A = \begin{pmatrix} 2 & -1 \\ \pi & 0 \end{pmatrix}\), \(B = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}\).

4. The matrix \(A = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\) creates a linear mapping (transformation) of the plane into itself. What are the images of the standard basis vectors \(e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\) and \(e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\)?

5. For the same basis vectors, find a matrix \(A\) such that \(Ae_1 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}\) and \(Ae_2 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}\). Is such a matrix unique?

6. Find the transpose \(A^t\) to the matrix \(A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}\).

7. Compute \(\det A\) and \(\text{tr}A\) for the matrix in the previous exercise.

8. What is the area of the parallelogram that has vectors \(a = \begin{pmatrix} -2 \\ 2 \end{pmatrix}\) and \(b = \begin{pmatrix} 3 \\ 1 \end{pmatrix}\) as its sides?

9. A matrix \(A\) sends the standard basis vectors \(e_1, e_2\) (see Problem 4) into two parallel (collinear) vectors. What is the determinant of \(A\)?
10. What does it mean that a matrix is non-singular? Write examples of a non-singular and of a singular matrices (with non-zero entries).

11. Find the inverse for the matrix $A$ in Problem 4.

12. Use the result of the previous problem and matrix algebra to solve the system

\[
\begin{align*}
3x_1 + x_2 &= 5 \\
x_1 + x_2 &= 7
\end{align*}
\]

13. Let $\lambda_1, \lambda_2$ be the eigenvalues of a matrix $A$. Prove that $\lambda_1 + \lambda_2 = \text{tr}A$, $\lambda_1 \times \lambda_2 = \det A$.

14. Can a matrix $A$ with real entries have eigenvalues $\lambda_1 = 2 + i$, $\lambda_2 = 3$?

15. Construct an example of a matrix that has eigenvalues $\lambda_1 = 2$, $\lambda_2 = 3$.

Find eigenvalues and corresponding eigenvectors of the following matrices:

16. $A = \begin{pmatrix} 5 & 3 \\ -4 & -2 \end{pmatrix}$

17. $A = \begin{pmatrix} 4 & 5 \\ -5 & -6 \end{pmatrix}$

18. $A = \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$