

Math151 Lecture Notes
Section 6.4 - The Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus, Part I

If f is continuous on $[a, b]$, then the function g is defined by

$$g(x) = \int_a^x f(t)dt \text{ for } a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b) and

$$g'(x) = f(x) \text{ or using Leibniz notation } \frac{d}{dx} \int_a^x f(t)dt = f(x).$$

Example 1: Find the derivative of the function $g(x) = \int_0^x \sqrt{1+t^2} dt$

Example 2: Find $\frac{d}{dx} \int_1^{x^4} \sec(t) dt$

The Fundamental Theorem of Calculus Part 1

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f , that is $F' = f$.

Example 3: Evaluate the integral $\int_{-2}^1 x^3 dx$

Example 4: Find the area under the parabola $y = x^2 + 1$ from 0 to 2.

Example 5: Find the area under the cosine curve from 0 to b where $0 \leq b \leq \frac{\pi}{2}$

Example 6: Find $\int (10x^4 + 3 \sec^2 x) dx$

Example 7: Evaluate $\int_0^3 (x^3 - 5x) dx$

Example 8: Evaluate $\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dt$

APPLICATIONS OF THE FUNDAMENTAL THEOREM

Change of position or displacement of a particle during the time interval $[t_1, t_2]$

$$\int_{t_1}^{t_2} v(t) dt = s(t_2) - s(t_1)$$

$$\text{Total Distance Traveled} = \int_{t_1}^{t_2} |v(t)| dt$$

Example 9: A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in m/s)

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$

(b) Find the distance traveled during this time period.

Table of Indefinite Integrals

$$\int c f(x) dx = c \int f(x) dx$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, (n \neq -1) \quad \int \frac{1}{x} dx = \ln |x| + C$$

$$\int e^x dx = e^x + C \quad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C \quad \int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C \quad \int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C \quad \int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{1}{x^2 + 1} dx = \tan^{-1} x + C \quad \int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + C$$