THE FOURTH MOMENT OF DIRICHLET L-FUNCTIONS

MATTHEW P. YOUNG, AMERICAN INSTITUTE OF MATHEMATICS/STANFORD

In 1927, Ingham proved the following asymptotic formula for the fourth power moment of the Riemann zeta function on the critical line:

$$\frac{1}{T}\int_0^T |\zeta(\tfrac{1}{2}+it)|^4 dt = a_4(\log T)^4 + O((\log T)^3),$$

where $a_4 = (2\pi^2)^{-1}$. In 1979, Heath-Brown improved this result by obtaining

$$\frac{1}{T}\int_0^T |\zeta(\tfrac{1}{2}+it)|^4 dt = \sum_{i=0}^{4} a_i(\log T)^i + O(T^{-\frac{1}{8}+\varepsilon}),$$

for certain explicitly computable constants a_i. The difficult part of extending Ingham's result to include the lower-order terms is asymptotically evaluating the off-diagonal terms.

The family of all primitive Dirichlet L-functions of modulus q is similar in some ways to the Riemann zeta function in t-aspect, but is more difficult to analyze. In 1981, Heath-Brown obtained an asymptotic formula for the fourth power moment

$$\sideset{}{^*}\sum_{\chi \,(\mathrm{mod}\ q)} |L(\frac{1}{2},\chi)|^4,$$

provided q does not have too many prime factors. Recently, Soundararajan was able to obtain the asymptotic formula for all q. I will discuss my recent work where I obtain the asymptotic formula

$$\frac{1}{q}\sideset{}{^*}\sum_{\chi \,(\mathrm{mod}\ q)} |L(\frac{1}{2},\chi)|^4 = \sum_{i=0}^{4} a_i'(\log q)^i + O(q^{-\frac{5}{512}+\varepsilon})$$

for prime q.