# UPDATES TO "SELF-SIMILAR GROUPS"

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**1.10.2. Free groups.** Mariya and Yaroslav Vorobets proved that the states of the automaton shown on the left-hand side of Figure 1.9 freely generate the rank 3 free group, thus have answered a question posed by S. Sidki.

Their preprint is available under

http://arxiv.org/abs/math.GR/0601231

They have also generalized the construction and defined two series of automata whose states freely generate the free groups of rank $n \geq 3$ and free products of $n$ copies of $C_2$ for $n \geq 3$. See their preprint at

http://arxiv.org/abs/math.GR/0604328

**3.8. The limit space $\mathcal{J}_G$ as a hyperbolic boundary.** Kevin Pilgrim presents a proof of Theorems 3.8.6 and 3.8.8 with a more geometric approach in

Kevin M. Pilgrim, *Julia sets as Gromov boundaries following V. Nekrashevych*, Topology Proceedings, **29** (2005), no. 1, 293–316.

**3.10.2. Examples.** Zoran Šunić and Rostislav Grigorchuk give an interesting interpretation of the group described in *Sierpiński gasket* (the group generated by the automaton from Figure 3.5). They show that this group models the *Hanoi towers* game. See their paper

Rostislav Grigorchuk and Zoran Šunić, *Asymptotic aspects of Schreier graphs and Hanoi Towers groups.* C. R. Math. Acad. Paris, **342** (2006), no. 8, 545–550.

and the preprint

http://arxiv.org/abs/math.GR/0601592

The same group can be described as a standard action of the iterated monodromy group of the antiholomorphic map $f(z) = \overline{z^2 - \frac{16}{27z}}$. This maps has four post-critical points (including infinity), which are fixed. Note that IMG $(f)$ is isomorphic as an abstract group to IMG $\left(z^2 - \frac{16}{27z}\right)$ and even their actions on the rooted trees are conjugate (since the second iterates of the functions coincide), but the limit dynamical systems are not conjugate (the rational function interchanges two post-critical points).

**6.5.2. Theorem of Thurston.** The preprint [BN05b] with solution of the J. H. Hubbard's "Twisted rabbit question" has been published

[BN06 ] Laurent Bartholdi and Volodymyr Nekrashevych, *Thurston equivalence of topological polynomials*, Acta Math., **197** (2006), 1–51.

*Date*: February 15, 2007.

(A preliminary version is available on arXiv.)

It also contains solutions of similar questions about some other quadratic polynomials.

**6.11. Quadratic polynomials.** The preprint [BN05a] is available now under

http://arxiv.org/abs/math.GR/0611177.

**6.12.1. The iterated monodromy group of $z^2 - 1$ and amenability.** The paper of L. Bartholdi and B. Virág [BV] is published:

[BV] Laurent Bartholdi and Bálint Virág, *Amenability via random walks*, Duke Math. J., **130** (2005), no. 1, 39–56.

**6.12.5. Groups of intermediate growth and** $\mathrm{IMG}(z^2 + i)$**.** The paper of K.-U. Bux and R. Perez [BP04] is published:

[BP06] Kai-Uwe Bux and Rodrigo Pérez, *On the growth of iterated monodromy groups*, in Topological and asymptotic aspects of group theory, 61–76, Contemp. Math., 394, Amer. Math. Soc., Providence, RI, 2006.