

Dynamical Systems and Chaos — Problem Set 1

Issued: 02.07 Due: 02.18

1.1. Tent map I. Sketch the graph of the tent map

$$T(x) = \begin{cases} 2x & 0 \leq x \leq 1/2 \\ 2 - 2x & 1/2 \leq x \leq 1 \end{cases}$$

on the unit interval $[0, 1]$. Use the graph of T^n to conclude that T has exactly 2^n periodic points of period n .

1.2. Tent map II. Show that the set of all periodic points of the tent map are dense in $[0, 1]$.

1.3. Fibonacci shift. Let Σ be the set of all sequences $x_1x_2\dots$ from $\{0, 1\}^\infty$ satisfying: if $x_k = 0$ then $x_{k+1} = 1$.

a) Show that the shift $\sigma(x_1x_2\dots) = x_2x_3\dots$ preserves the set Σ .

b) Show that Σ is closed in $\{0, 1\}^\infty$.

c) How many fixed points are there for σ , σ^2 and σ^3 in Σ .

c) Find a recursive formula for the number of fixed points of $\sigma^n : \Sigma \rightarrow \Sigma$ in terms of the number of fixed points of σ^{n-1} and σ^{n-2} .

1.4. Topological conjugacy. Construct a piecewise linear map on $[0, 1]$ which is topologically conjugate to $F(x) = 4x^3 - 3x$ on $[-1, 1]$.