Texas A&M University
Department of Mathematics
Volodymyr Nekrashevych

Spring 2008

Dynamical Systems and Chaos — Problem Set 2
Issued: 03.06 Due: 03.24

2.1. Linear maps Describe the dynamics of the liear maps whose matrix representation is

a) $\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$,

b) $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$,

c) $\begin{pmatrix} 2 & 1 \\ 0 & \frac{1}{2} \end{pmatrix}$,

d) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$,

e) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$,

f) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$.

2.2. The full shift. Let $\Sigma_2 = \{0,1\}^{\mathbb{Z}}$ be the full bilateral shift space. Let $\sigma : \Sigma_2 \longrightarrow \Sigma_2$ be the shift.

a) Construct a dense orbit for σ.

b) Prove that periodic points are dense for σ.

c) Let $w = \ldots x_{-1} \, . \, x_0 x_1 \ldots$. Describe the sets $W^s(w)$ and $W^u(w)$.

2.3. One sided vs two sided. Prove that the map

$$\Phi(x_0x_1x_2\ldots) = (\ldots x_5x_3x_1 \, . \, x_0x_2x_4\ldots)$$

is a homeomorphism between the spaces of one-sided and two-sided sequences over same alphabet X.

2.4. Angle doubling on the torus. Consider the map A from the torus to itself defined by $A(x,y) = (2x, 2y)$.

a) Show that A is a four-to-one self-map of the torus.

b) Prove that periodic points are dense for this map.

c) Prove that eventually fixed points are dense.