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Dynamical Systems and Chaos — Problem Set 3
Issued: 03.31 Due: 04.07

3.1. The Hénon Attractor. Consider the diffeomorphism of the plane given by

$$\begin{aligned} x_1 &= 1 + y - 1.4x^2 \\ y_1 &= 0.3x. \end{aligned}$$

(a) Consider the quadrilateral Q whose vertices are given by the four points $(-1.33, 0.42)$, $(1.32, 0.133)$, $1.245, -0.14)$, and $(-1.06, -0.5)$. Prove that $F(Q) \subset Q$.

(b) Using a computer, compute $10,000$ iterates of a point in Q. Plot the last $9,000$ points. Note that the resulting picutre appears to be the same no matter which random initial point is chosen.

3.2. Consider the diffeomorphism F of the plane given in polar coordinates by

$$\begin{aligned} r_1 &= \lambda r + \beta r^3 \\ \theta_1 &= \theta + \tfrac{2\pi}{n} + \epsilon \sin(n\theta) \end{aligned}$$

where $\epsilon > 0$ is small, $\lambda > 1$ and $\beta < 0$.

a. Identify and classify all periodic points of F.

b. Show that the circle γ given by $r = \sqrt{(1-\lambda)/\beta}$ is invariant under F.

c. Identify and sketch the stable and unstable manifolds of the saddle points of F.