

M147, Fall 2009, Exam 1 Name _____

Note for Fall 2010. This is Exam 1 for M147, Fall 2009, and the exam this year will have the same format (ten multiple choice questions and five problems for which work is to be written out). This exam covered two sections that will not be on Exam 1 for Fall 2010, Sections 4.1 and 4.2. In light of this, you should skip problems 8, 9, 10, 14, and 15.

Calculators are not allowed on the exam. The first ten problems are multiple choice. Written out work on these problems will not be checked, so take care in marking your answers. For Problems 11-15 unjustified answers will not receive credit.

1. [5 pts] Which of the following functions corresponds with the graph in Figure 1?

(a) $y = \log_2 x$

(b) $y = \log_{\frac{1}{2}} x$

(c) $y = 2^x$

(d) $y = \left(\frac{1}{2}\right)^x$

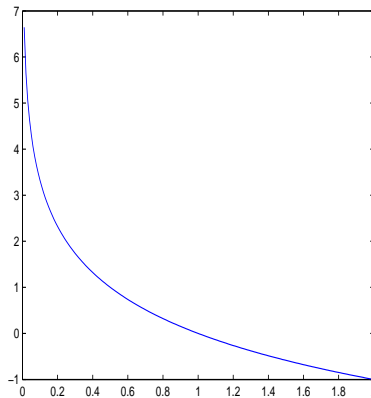


Figure 1: Figure for Problem 1.

2. [5 pts] Use a logarithmic transformation to find a linear relationship between (appropriate transformations of) x and y if

$$y = 4x^5.$$

(a) $\log y = 4x + \log 5$

(b) $\log y = 5x + \log 4$

(c) $\log y = 5 \log x + \log 4$

(d) $\log y = 4 \log x + \log 5$

(e) None of the above

3. [5 pts] Compute the limit

$$\lim_{x \rightarrow 1^-} \frac{x}{1-x}.$$

- (a) $+\infty$
- (b) $-\infty$
- (c) 0
- (d) 1
- (e) None of the above

4. [5 pts] Compute the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 4} - 2}{x^2}.$$

- (a) 0
- (b) 1
- (c) $\frac{1}{2}$
- (d) $\frac{1}{4}$
- (e) None of the above

5. [5 pts] Compute the limit

$$\lim_{x \rightarrow \infty} \frac{3x^3 + 7x^2 - 1}{x - 6x^2 - 6x^3}.$$

- (a) 3
- (b) $-\frac{1}{2}$
- (c) 0
- (d) $+\infty$
- (e) None of the above

6. [5 pts] Compute the limit

$$\lim_{x \rightarrow 0} \frac{\sin 2x \sin 5x}{x^2}.$$

- (a) 0
- (b) 1
- (c) 10
- (d) DNE
- (e) None of the above

7. [5 pts] Find a value c so that the following function is continuous for all values of x :

$$f(x) = \begin{cases} 1 - x^2 & x \leq 2 \\ cx + 1 & x > 2 \end{cases}.$$

- (a) 0
- (b) 1
- (c) -1
- (d) 2
- (e) None of the above

8. [5 pts] Which of the following statements is true?

- (a) If $f(x)$ is continuous then $f(x)$ is differentiable
- (b) If $f(x)$ is differentiable then $f(x)$ is continuous
- (c) If $f(x)$ is not differentiable then $f(x)$ is not continuous
- (d) All of (a), (b), and (c)
- (e) None of (a), (b), and (c)

9. [5 pts] Compute $f'(0)$, where $f(x)$ is given by

$$f(x) = \begin{cases} \sin x & x \leq 0 \\ x & x > 0 \end{cases}.$$

- (a) 1
- (b) -1
- (c) DNE
- (d) 0
- (e) None of the above

10. [5 pts] Compute $f'(0)$, where $f(x)$ is given by

$$f(x) = \begin{cases} \frac{x^3}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

- (a) 1
- (b) -1
- (c) DNE
- (d) 0
- (e) None of the above

11. [10 pts] Given the doublelog plot in Figure 2, find a functional relationship between x and y .

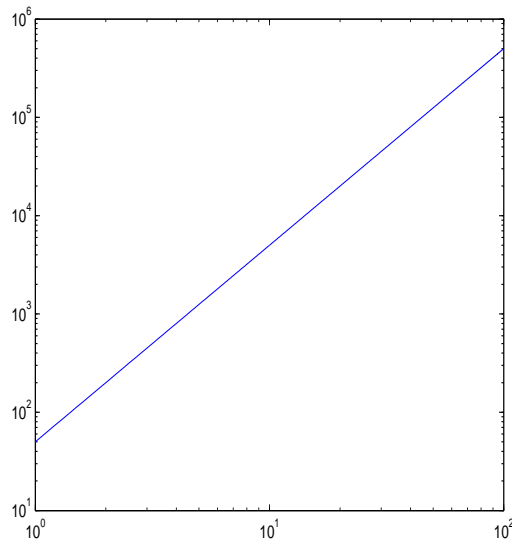


Figure 2: Figure for Problem 11.

12. [5 pts each] Determine whether or not the following functions are continuous at $x = 0$. In each case, explain why or why not.

12a.

$$f(x) = \begin{cases} 2x^2 + 1, & x \leq 0 \\ 2x^2 - 1, & x > 0 \end{cases}.$$

12b.

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

13. [10 pts] Use the bisection method to approximate a root of

$$x^5 + x - 3 = 0$$

with a maximum error of $\frac{1}{3}$.

14. [10 pts] Find an equation for the line that is tangent to the given curve at $x = 2$.

$$y = \frac{1}{4}x^4 - x^3.$$

15. [10 pts] A car moves along a straight road. Its location at time t is given by

$$s(t) = \frac{1}{6}t^3,$$

where t is measured in hours and $s(t)$ is measured in kilometers.

15a. Graph $s(t)$ for $0 \leq t \leq 3$.

15b. Find the average velocity of the car between $t = 0$ and $t = 3$. Illustrate the average velocity on your graph of $s(t)$.

15c. Find the instantaneous velocity of the car at time $t = 1$. Illustrate the instantaneous velocity on the graph of $s(t)$.