M147, Fall 2009, Exam 2 Name______________________

Note for Fall 2010. This is Exam 2 for M147, Fall 2009, and the exam this year will have the same format (ten multiple choice questions and five problems for which work is to be written out). This exam covered two sections that will not be on Exam 1 for Fall 2010, Sections 5.1 and 5.2. In light of this, you should skip problems 13, 14, and 15. On the other hand, you should go back and work problems 8, 9, 10, 14, and 15 from last year's Exam 1.

Calculators are not allowed on the exam. The first ten problems are multiple choice. Written out work on these problems will not be checked, so take care in marking your answers. For Problems 11-15 unjustified answers will not receive credit.

1. [5 pts] Compute the derivative of the given function.

$$f(x) = e^{2x}\sin 3x.$$

(a) $f'(x) = \frac{1}{2}e^{2x}\sin 3x + \frac{1}{3}e^{2x}\cos 3x$

(b) $f'(x) = \frac{1}{2}e^{2x}\sin 3x - \frac{1}{3}e^{2x}\cos 3x$

(c) $f'(x) = 2e^{2x}\sin 3x + 3e^{2x}\cos 3x$

(d) $f'(x) = 2e^{2x}\sin 3x - 3e^{2x}\cos 3x$

(e) None of the above

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2. [5 pts] Compute the derivative of the given function.

$$f(x) = \frac{\sqrt{x}-1}{\sqrt{x}+1}.$$

(a) $f'(x) = \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$

(b) $f'(x) = \frac{1}{(\sqrt{x}+1)^2}$

(c) $f'(x) = -\frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$

(d) $f'(x) = -\frac{1}{(\sqrt{x}+1)^2}$

(e) None of the above

3. [5 pts] Compute the derivative of the given function.

$$f(x) = \sqrt{1 + \sin(x^2)}.$$

(a) $f'(x) = \frac{x}{\sqrt{1+\sin(x^2)}}$

(b) $f'(x) = \frac{1}{2\sqrt{1+\sin(x^2)}}$

(c) $f'(x) = \frac{\cos(x^2)}{2\sqrt{1+\sin(x^2)}}$

(d) $f'(x) = \frac{x\cos(x^2)}{\sqrt{1+\sin(x^2)}}$

(e) None of the above

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4. [5 pts] Compute the derivative of the given function.

$$f(x) = 2^{\log_5 x}.$$

(a) $f'(x) = 2^{\log_5 x} \ln 2$

(b) $f'(x) = 2^{\log_5 x} \frac{\ln 2}{x}$

(c) $f'(x) = 2^{\log_5 x} \frac{\ln 2}{x \ln 5}$

(d) $f'(x) = 2^{\log_5 x} \frac{(\ln 2)(\ln 5)}{x}$

(e) None of the above

5. [5 pts] Evaluate the expression

$$\sin(\cos^{-1}(\frac{4}{5})).$$

(a) $\frac{1}{2}$

(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{3}{4}$

(d) $\frac{3}{5}$

(e) None of the above

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6. [5 pts] Let

$$f(x) = 3 + x + e^x,$$

and compute $\frac{df^{-1}}{dx}(4)$.

(a) -1

(b) 0

(c) 1

(d) 2

(e) None of the above

7. [5 pts] Compute the derivative of the given function.

$$f(x) = x^{\cos x}, \quad x \geq 0.$$

(a) $f'(x) = x^{\cos x}(\frac{\cos x}{x} - \sin x \ln x)$

(b) $f'(x) = -x \sin x + \cos x$

(c) $f'(x) = x^{\cos x}(-x \sin x + \cos x)$

(d) $f'(x) = \frac{\cos x}{x} - \sin x \ln x$

(e) None of the above

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8. [5 pts] Find an equation for the line that is tangent to the curve described by

$$x^2 - xy + y^3 = 8$$

at the point $(2, 2)$.

(a) $y - 2 = 8(x - 2)$

(b) $y - 2 = \frac{1}{8}(x - 2)$

(c) $y - 2 = -\frac{1}{5}(x - 2)$

(d) $y - 2 = -5(x - 2)$

(e) None of the above

9. [5 pts] Let $h(x) = \frac{f(x)}{g(x)}$, and suppose $f(7) = -4$, $f'(7) = 3$, $g(7) = 1$, and $g'(7) = 8$. Find $h'(7)$.

(a) $h'(7) = -29$

(b) $h'(7) = 29$

(c) $h'(7) = -35$

(d) $h'(7) = 35$

(e) None of the above

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10. [5 pts] Use a linear approximation to estimate a value for

$$\tan^{-1} .99.$$

(a) $\frac{\pi}{4} + .005$

(b) $\frac{\pi}{4} - .005$

(c) $1 + .005$

(d) $1 - .005$

(e) None of the above

11. [10 pts] A balloon is released from the ground 4 meters from a stationary observer. If the balloon rises vertically at a constant rate of 1 meter per second, how fast is the distance between the observer and the balloon changing when the balloon has risen 3 meters?

12. [10 pts] Find $\frac{d^2y}{dx^2}$ when

$$x^4 + y^4 = 1.$$

13. [10 pts] For the function

$$f(x) = \frac{x}{(x-1)^2}; \quad x \neq 1,$$

determine the intervals on which $f(x)$ is decreasing and the intervals on which $f(x)$ is increasing.

14. [10 pts] Suppose that for a given function $f(x)$ you know the first derivative is

$$f'(x) = \frac{x^2+1}{x^2-1}; \quad x \neq \pm 1.$$

Determine the intervals on which $f(x)$ is concave down and the intervals on which $f(x)$ is concave up.

15. [10 pts] Suppose $f(x)$ is twice differentiable in an open interval containing the point $x = c > 0$, and has a local maximum at the same point, $f(c) > 0$. Show that the function $g(x) = \ln f(x)$ has a local maximum at $x = c$.