M611 Fall 2019, Assignment 8, due Friday Nov. 1

1. [10 pts] Find the Green's function for Laplace's equation on the quarter plane $U = \mathbb{R}_+ \times \mathbb{R}_+$. Use your Green's function to solve Laplace's equation

$$\begin{aligned} \Delta u &= 0 \quad \text{in } U, \\ u &= g \quad \text{on } \partial U. \end{aligned}$$

You need not prove that $u(\vec{x})$ defined this way is a solution.

2. [10 pts] Find the Green's function for Laplace's equation on the infinite wedge

$$U = \{(r, \theta) : 0 < r < \infty, 0 < \theta < \frac{\pi}{3}\}.$$

In this case you only need to find $G(\vec{x}, \vec{y})$; in particular, you do not need to use it to write down a solution to Laplace's equation.

3. [10 pts] (**Evans 2.5.7.**) Use Poisson's formula for the ball to prove

$$r^{n-2} \frac{r - |\vec{x}|}{(r + |\vec{x}|)^{n-1}} u(0) \leq u(\vec{x}) \leq r^{n-2} \frac{r + |\vec{x}|}{(r - |\vec{x}|)^{n-1}} u(0)$$

whenever u is positive and harmonic in $B^o(0, r)$. This is an explicit form of Harnack's inequality.

4. [10 pts] (**Evans 2.5.8.**) Prove Theorem 15 of Section 2.2.4. (Hint: Since $u \equiv 1$ solves (44) for $g \equiv 1$, the theory automatically implies

$$\int_{\partial B(0,1)} K(\vec{x}, \vec{y}) dS_y = 1$$

for each $\vec{x} \in B^o(0, 1)$.)