M612 Spring 2020, Assignment 7, due Fri., Mar. 20

1. [10 pts] Show that if $u \in H_0^1(U)$, with $U = B^o(0,1) \subset \mathbb{R}^n$, then

$$\Big| \int_U u(\vec{x}) d\vec{x} \Big| \le \frac{\alpha(n)^{\frac{1}{2}}}{\sqrt{n(n+2)}} \|Du\|_{L^2(U)},$$

where as usual $\alpha(n)$ denotes the volume of a unit ball in $\mathbb{R}^n$.

2. [10 pts] Let $I = (a,b) \subset \mathbb{R}$ be an interval, and suppose $1 < p < \infty$. Show that given any $\epsilon > 0$ there exists a constant β_ϵ, depending on ϵ and I, so that

$$|u(b)| \le \beta_\epsilon \|u\|_{L^p(I)} + \epsilon \|u'\|_{L^p(I)},$$

for all $u \in W^{1,p}(I)$.

3. [10 pts] (**Evans 5.10.11.**) Suppose U is connected and $u \in W^{1,p}(U)$ satisfies

$$Du = 0 \quad \text{a.e. in } U.$$

Prove u is constant a.e. in U.

Note. This problem is primarily intended to make you look at Theorem 5.8.1, which we won't discuss in class.

4. [10 pts] (**Evans 5.10.12.**) Show by example that if we have $\|D^h u\|_{L^1(V)} \le C$ for all $0 < |h| < \frac{1}{2}\text{dist}\,(V, \partial U)$, it does not necessarily follow that $u \in W^{1,1}(V)$.

Note. If you have trouble thinking of an example, peruse old homework problems.