M642 Assignment 5, due Friday Feb. 22

1. [10 pts] Solve the following:

a. (**Keener Problem 4.3.1.**) Find the adjoint operator L^* and its domain for the operator

$$Lu = u'' + a(x)u' + b(x)u; \quad u(0) = u'(1), u(1) = u'(0).$$

Assume $a(x)$ is continuously differentiable and $b(x)$ is continuous on the interval $[0, 1]$.

b. (**Keener Problem 4.3.3.**) Find the adjoint operator L^* and its domain for

$$\begin{aligned} Lu &= u'' + 4u' - 3u \\ u'(0) + 4u(0) &= 0 \\ u'(1) + 4u(1) &= 0. \end{aligned}$$

2. [10 pts] Find the adjoint operator for

$$\begin{aligned} Lu &= (u'' - V(x)u)'' = f(x) \\ u'(0) &= u'(1) = 0 \\ u'''(0) &= u'''(1) = 0. \end{aligned}$$

Is this operator self-adjoint?

3. [10 pts] Consider the eigenvalue problem

$$\begin{aligned} Lu &= (u'' - V(x)u)'' = \lambda u \\ u'(0) &= u'(1) = 0 \\ u'''(0) &= u'''(1) = 0, \end{aligned}$$

for which we assume $V'(0) = V'(1) = 0$.

a. Set $v(x) = \int_0^x u(y)dy$ and write down the eigenvalue problem (including boundary conditions) for v, assuming $\lambda \neq 0$. (Keep in mind that your boundary conditions should not involve derivatives as high as the order of your equation.)

b. Let $\mathcal{L}$ denote your operator on v (again, including boundary conditions) and compute $\mathcal{L}^*$. Is $\mathcal{L}$ self-adjoint?

4. [10 pts] (**Keener Problem 4.3.9.**) Find solvability conditions for the equation

$$u'' + u = f(x); \quad u(0) - u(2\pi) = \alpha, u'(0) - u'(2\pi) = -\beta.$$

5. [10 pts] (**Keener Problem 4.3.12.**) Find solvability conditions for the equation

$$(xu')' = f(x); \quad u'(1) = \alpha, xu'(x)\Big|_{x=0} = \beta.$$

Note. Although this problem does not satisfy all of our assumptions for the Fredholm Alternative, you can apply it in this case.