Dominique Picard

Title: A 'Frame-work' in Learning Theory (joint work with Gérard Kerkyacharian)

Abstract: We investigate the problem of learning a bounded function. In this context, we shall have as our goal to obtain estimations to f_ρ with the error measured in the $L_2(\mathcal{X}, \rho_\mathcal{X})$ norm, and more precisely, given any $\eta > 0$, if $\hat{f}$ is an estimator of f_ρ we want to have good bounds for the following measure of risk :

$$\sup_{\rho \in M(\Theta)} \rho\{\|\hat{f} - f_\rho\| > \eta\}$$

where here $M(\Theta)$ indicates that f_ρ belongs to Θ. These bounds typically depends on the metric properties of Θ, and the best performances generally are obtained by estimators minimizing the emprical risk, over some particular subspaces. This may lead to very complicated calculations if one additionally wants to have 'universal' estimators (i.e. estimators attaining the best bounds for a large set of spaces Θ.)

In some cases, this minimization reduces to a thresholding algorithm on some coefficients. We concentrate on this approach, and prove that, under a 'near identity property' of a matrix associated to the problem, a thresholding estimate has almost universal and optimal exponential bounds.

The next step consists in showing that this 'near identity property' can be verified in a large variety of situations. In the learning problem, the observations are of the form $(X_1, Y_1), \ldots, (X_n, Y_n)$. Depending on the dimension of the space where the X_i's are lying, the difficulty is not the same. In the case where the space of the X_i's is a compact set of $\mathbb{R}$, we give a natural and almost straightforward solution. In the d-dimensional case, we provide a way to obtain such a condition with the associated estimators, using frames. However, this may lead to a reduction of the efficiency of the method, or to additional assumptions on the model. We also provide an enhancement of the procedure using empirical bayes methods.

References

[1] DeVore, R., Kerkyacharian, G., Picard, D. and Temlyakov, V. *Approximations methods for supervised learning.*, Found. Comp. Math. (2006) **6**, 3–58.

[2] Kerkyacharian, G., and Picard, D. *Thresholding in learning theory*, Constr. Approx. (2007) **26**, 173–203