

2.5.5 (a)

$$\frac{d^2\phi}{d\theta^2} = -\lambda \phi \text{ subject to } \frac{d\phi}{d\theta}(0) = 0 \text{ and } \phi(\pi/2) = 0$$

$$\Rightarrow \cos \sqrt{\lambda} \theta = \cos \sqrt{\lambda} \frac{\pi}{2} = 0 \text{ implies that } \lambda = (2n-1)^2$$

The boundedness condition at $r=0$ yields $G(r) = r^{2n-1}$

$$u(r, \theta) = \sum_{n=1}^{\infty} A_n r^{2n-1} \cos((2n-1)\theta)$$

$$f(\theta) = \sum_{n=1}^{\infty} A_n \cos((2n-1)\theta) \text{ or } A_n = \frac{4}{\pi} \int_0^{\pi/2} f(\theta) \cos((2n-1)\theta) d\theta$$

(d)

$$\frac{d^2\phi}{d\theta^2} = -\lambda \phi \quad \frac{d\phi}{d\theta}(0) = 0 \quad \text{and} \quad \frac{d\phi}{d\theta}(\pi) = 0$$

$$\Rightarrow \frac{d\phi(\theta)}{d\theta} = (C_2 \sin(\sqrt{\lambda}\theta)) \sqrt{\lambda} = 0 \Rightarrow \sqrt{\lambda} = 2n.$$

$$G(r) = Cr^{2n}$$

$$u(r, \theta) = \sum_{n=1}^{\infty} A_n r^{2n} \cos(2n\theta)$$

$$\text{Since } \frac{\partial u}{\partial r}(1, \theta) = g(\theta), \Rightarrow \frac{\partial u}{\partial r}(1, \theta) = \sum_{n=1}^{\infty} A_n \cos(2n\theta) (2n)r^{2n-1} = g(\theta)$$

$$A_n = \frac{2}{n\pi} \int_0^{\pi/2} g(\theta) \cos(2n\theta) d\theta$$

2.5.7. (a)

$$\phi(0) = 0, \phi(\pi/3) = 0 \text{ implies } \phi(\theta) = C_2 \sin(3n\theta), \sqrt{\lambda} = 3n$$

$$G(r) = Cr^{3n}$$

$$u(r, \theta) = \sum_{n=1}^{\infty} A_n r^{3n} \sin(3n\theta)$$

$$u(a, \theta) = f(\theta) = \sum_{n=1}^{\infty} A_n a^{3n} \sin(3n\theta)$$

$$A_n a^{3n} = \frac{6}{\pi} \int_0^{\pi/3} f(\theta) \sin(3n\theta) d\theta$$

②