1. Find the radius and interval of convergence for the following power series.

(a) \[ \sum_{n=1}^{\infty} \frac{(x - 1)^n}{3^n} \]

(b) \[ \sum_{n=2}^{\infty} \frac{(2x - 1)^n}{4^n \ln n} \]

(c) \[ \sum_{n=1}^{\infty} \frac{(3x - 1)^{2n}}{n \sqrt{n} 5^n} \]
(d) \[ \sum_{n=1}^{\infty} (\sqrt{n} + 1 - \sqrt{n})(x - 3)^n \]

(e) \[ \sum_{n=1}^{\infty} [2 + (-1)^n](x + 1)^n \]

(f) \[ \sum_{n=1}^{\infty} \frac{x^n}{n \ln n} \]
2. Find the radius of convergence for the power series \( \sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdots (3n - 1)}{2 \cdot 4 \cdots (2n)} x^n \).

3. Expand the function \( f(x) = \frac{1}{4+3x} \) in a power series centered at 0, and determine the values for \( x \) for which the expansion is valid.

4. Expand \( \frac{1}{1 - x} \) in a power series centered at 5, and determine its interval of convergence.
5. Find a power series representation for \( f(x) = \ln(3 + x^2) \).

6. (a) Find a power series representation for \( f(x) = \frac{1}{(1 + x)^2} \). What is the radius of convergence?

(b) Find the sum of the series
\[
\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \cdots
\]
7. Estimate \( \int_{0}^{0.4} \frac{t}{1 + t^8} \, dt \) correct to 8 decimal places.

8. (a) Find the interval of convergence of the power series \( \sum_{n=1}^{\infty} nx^n \) and \( \sum_{n=1}^{\infty} n^2 x^n \).

(b) Find a formula for the sum of the two power series.

(c) Find the sum of each of the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{3^n} \) and \( \sum_{n=1}^{\infty} \frac{n^2}{2^n} \).
9. Suppose we know that for an unknown sequence \((c_n)\) the series \(\sum_{n=0}^{\infty} (-1)^n 3^n c_n\) converges, but the series \(\sum_{n=0}^{\infty} 5^n c_n\) diverges. What can be said about the following series?

(a) \(\sum_{n=0}^{\infty} c_n\)

(b) \(\sum_{n=0}^{\infty} (-1)^n 6^n c_n\)

(c) \(\sum_{n=0}^{\infty} 3^n c_n\)

(d) \(\sum_{n=1}^{\infty} n(n-1) 2^n c_n\)

(e) \(\sum_{n=0}^{\infty} (-\pi)^n \frac{c_n}{n + 1}\)