Week in Review, Sections 7.5-7.6

1. A pair of fair dice are tossed. Suppose we know that the sum of the two numbers is 9. What is the probability that their product is 20?

2. Use the tree diagram to find the required probabilities.

   (a) $P(C|B)$

   (b) $P(A \cap B)$

   (c) $P(D)$

   (d) $P(B \cup D)$

   (e) $P(A|C)$
3. In a box are 4 pieces of paper numbered 1 through 4. An experiment consists of pulling two slips of paper out of the box in succession without replacement. What is the probability that

(a) One of the numbers is a 2 given that the sum of the two numbers is less than or equal to 4?

(b) The sum of the two numbers is 5, given that the first number drawn is a 4 or a 3?

4. Suppose for two events $A$ and $B$ that $P(A) = 0.3$ and $P(B^c) = 0.8$. If it is known that $A$ and $B$ are independent events, what is $P(A^c \cup B^c)$?

5. Given that $A$ and $B$ are independent with $P(A \cup B) = 0.8$ and $P(B^c) = 0.3$, find $P(A)$. 
6. A certain university in Texas admitted 35% of the applicants from Texas, 15% of the applicants from other U.S. states, and 11% of international applicants. There were a total of 10000 applicants to this university. 7200 were from Texas, 2000 were from other U.S. states, and 800 were international applicants.

(a) What is the probability that an applicant is from Texas and is admitted to the university?

(b) What is the probability that an applicant is not admitted to the university?

(c) What is the probability that an applicant from a U.S. state other than Texas is not admitted to the university?

(d) What is the probability that an applicant who is admitted to the university is an international applicant?
7. An experiment consists of drawing a card, then another, at random, from a well-shuffled standard deck of 52 playing cards. Find the probability that:

(a) the second card drawn is a queen.

(b) the first card was a queen given that the second card is a queen.

8. A survey conducted by an independent agency for the National Lung Society found that of 2000 women, 680 were heavy smokers and 50 had emphysema. Of those who had emphysema, 42 were also heavy smokers. Using the data in this survey, determine whether the events “being a heavy smoker” and “having emphysema” are independent events.
9. Urn A contains 2 white and 4 red balls. Urn B contains 1 white and 1 red ball. A ball is first drawn from urn A, and then put in urn B. A second ball is then drawn from urn B.

(a) What is the probability that the ball drawn from urn B is white?

(b) What is the probability that the transferred ball was white given that a white ball is drawn from urn B?

10. A city police department reported that, of all vehicles stolen, 64% were stolen by professionals, whereas 36% were stolen by amateurs. Of those vehicles stolen by professionals, 24% were recovered within 48 hrs, 16% were recovered after 48 hrs, and 60% were never recovered. Of those vehicles stolen by amateurs, 38% were recovered within 48 hrs, 58% were recovered after 48 hrs, and 4% were never recovered. Suppose a stolen vehicle has been recovered within 48 hours.
(a) What is the probability that it had been stolen by amateurs?

(b) What is the probability that it had been stolen by professionals?

11. During a given winter, 23% of the population received a flu shot, and of those, 1% got the flu. 12% of those who did not receive a flu shot got the flu.

(a) What is the probability that a person got the flu?

(b) What is the probability that a person who did not get the flu received the flu shot?
12. **(Bertrand’s Box Problem)** There are three boxes: a box containing two gold coins, a box with two silver coins, and a box with one of each. After choosing a box at random and withdrawing one coin at random that happens to be a gold coin, what is the probability that the other coin is gold?

13. **(Su, Francis E., et al. “Medical Tests and Bayes’ Theorem” Math Fun Facts)** Suppose that you are worried that you might have a rare disease. You decide to get tested, and suppose that the testing methods for this disease are correct 99 percent of the time (in other words, if you have the disease, it shows that you do with 99 percent probability, and if you don’t have the disease, it shows that you do not with 99 percent probability). Suppose this disease is actually quite rare, occurring randomly in the general population in only one of every 10,000 people. If your test results come back positive, what are your chances that you actually have the disease?