Week in Review, Sections 6.1-6.2

1. Let $A = \{1, 3, 5, 7\}$, $B = \{3, 5\}$, and $C = \{3, 7, 5, 1\}$. Determine whether the following statements are True or False.

(a) TRUE  FALSE  $A = C$

(b) TRUE  FALSE  $A \subseteq C$ (proper subset)  ($\text{In fact } A \subseteq C$)

(c) TRUE  FALSE  $B \subseteq A$

(d) TRUE  FALSE  $3 \in B$

(e) TRUE  FALSE  $\emptyset \in C$  ($\emptyset \subseteq C$  $\emptyset \not\subseteq C$)

(f) TRUE  FALSE  $\{1, 5\} \subseteq A$  ($\text{also } \{1, 5\} \subseteq A$)

(g) TRUE  FALSE  $5 \in B$  (However, $5 \in B$, $\{5\} \subseteq B$)

(h) TRUE  FALSE  $\{1\} \in C$  (unless you have $C = \{\ldots, \{1\}, \ldots\}$)
2. Given the sets \( R = \{x | x \) is a letter of the word “care”\}, \( S = \{x | x \) is a letter of the word “acre”\}, \\( T = \{x | x \) is a letter of the word “car”\}, \( V = \{x | x \) is a letter of the word “are”\}, \( W = \{x | x \) is a letter of the word “book”\}, fill in the blanks.

\[
\begin{align*}
\text{\( R \)} &= \{c, a, r, e\} \\
\text{\( S \)} &= \{a, c, v, e\} \\
\text{\( T \)} &= \{c, a, r\} \\
\text{\( W \)} &= \{b, o, k\} \\
\text{\( S \cup T \)} &= \{c, a, r\} \\
\text{\( T \cap V = \emptyset \)} &\text{ \(\implies \emptyset \subseteq \emptyset\)} \\
\text{\( S \cap W = \emptyset \)} &\text{ \(\text{\(s\) and \(w\) are disjoint}}\)
\end{align*}
\]

\[
\begin{align*}
\text{\( S \cup T = S \)} &\text{ \(\implies \emptyset \subseteq \emptyset\)} \\
\text{\( S \cap T = \emptyset \)} &\text{ \(\subseteq \emptyset\)} \\
\text{\( T \cup V = \emptyset \)} &\text{ \(\subseteq \emptyset\)}
\end{align*}
\]
3. Simplify the following.

\[(A^c \cup B)^c = (\overline{A^c} \cap \overline{B}) = A \cap B^c\]

\[(A \cap B^c)^c = \overline{A} \cup B\]

\[(A \cap B \cap C)^c = A^c \cup B^c \cup C^c\]

\[(A \cup B \cup C)^c = \overline{A} \cap B^c \cap C\]

\[\{ (A \cup B)^c = A^c \cap B^c \}
\[ (A \cap B)^c = A^c \cup B^c \]
\[ (A^c)^c = A \]
4. In the Venn diagrams below shade the following sets: $A \cap B^c$, $(A \cap B)^c$, $A^c \cap B^c$, $A \cup B^c$. Elements in $A$, but not in $B$ shade $\frac{\text{not in } B}{\text{(and)}} = A \cup B^c$ $(A \cap B)^c$ $A^c \cap B^c = (A \cup B)^c$ $A \cup B^c$
5. In the Venn diagrams below shade the following sets: \( A \cap B \cap C \), \( (A \cap B) \cup C \), \( A \cap B \cup C^c \), \( A \cup B^c \).

\[ A = \{a, b, c, d\}, \quad B = \{b, d, e, f\}, \quad C = \{c, d, f, g\} \]
\[ A \cap B = \{b, d\}, \quad (A \cap B) \cup C = \{a, c, d, e, f, g\} \]
\[ (A \cap B)^c \cup C = \{a, c, g, f, e, h, d\} \]
6. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, A = \{x \in U | x \text{ is a multiple of } 3\}, B = \{2, 3, 7, 10\}, \text{ and } C = \{1, 4, 5, 6, 7, 8\}$. Find the following sets.

(a) $A \cap C = \{6\}$

(b) $(B \cup C)^c$

$B \cup C = \{2, 3, 7, 10, 1, 4, 5, 8, 6\}$

$(B \cup C)^c = \{9\}$

(c) $B \cap (A \cup C)$

$A \cup C = \{1, 2, 3, 6, 9, 1, 4, 5, 7, 8\}$

$B \cap (A \cup C) = \{3, 7\}$

(d) $A^c \cup (B \cap C)$

$A^c = \{1, 2, 4, 5, 7, 8, 10\}$

$B \cap C = \{3, 6\}$

$A^c \cup (B \cap C) = \{1, 2, 4, 5, 7, 8, 10\} \cup \{3, 6\} = \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$

(e) $(C^c \cap (A \cup B))^c$

$C^c = \{2, 3, 4, 5, 9, 10\}$

$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$(C^c \cap (A \cup B))^c = \{2, 3, 4, 5, 9, 10\} \cap \{3, 6, 9, 2, 7, 10\}^c$

$= \{2, 3, 9, 10\} \cap \{1, 4, 5, 8\} = \emptyset$
7. The customers of a coffee shop are surveyed. Let $U$ denote the set of all the customers surveyed, and let $A = \{x \in U \mid x \text{ is female}\}$, $B = \{x \in U \mid x \text{ is male}\}$, $C = \{x \in U \mid x \text{ has cookie with their coffee}\}$, and $D = \{x \in U \mid x \text{ drinks water before coffee}\}$. Write the set that represents each statement.

(a) The coffee shop customers who are men and don’t have cookie with their coffee.

\[ B \cap C^c \]

(b) The coffee shop customers who are women and drink water before their coffee but don’t have cookie with their coffee.

\[ A \cap D \cap C^c = (A \cap D) \cap C^c = A \cap (D \cap C^c) \]

U: Keywords

- “or”
- “at least”

\[ A \cup (B \cap C) \neq (A \cup B) \cap C \]
8. In the town of Springfield, 3090 people are surveyed, and it is found that 1030 jog but don’t go to gym, 1110 go to gym but don’t jog, and 260 neither jog nor go to gym.

(a) How many both jog and go to the gym?
\[ n(\text{jog} \cap \text{gym}) = 3090 - (260 + 1030 + 1110) = 690 \]

(b) How many jog?
\[ n(\text{jog}) = 1030 + 690 = 1720 \]
9. Let $U$ be the universal set of all students at Texas A\&M.
   $C = \{x \in U | x \text{ listens to country music}\}$
   $R = \{x \in U | x \text{ listens to rock music}\}$
   $P = \{x \in U | x \text{ listens to pop music}\}$

Write the following sets in words:

(a) $C \cap P^c$ \quad The set of students who listen to country music, but not pop.

(b) $R^c \cup P$ \quad The set of students who do not listen to rock or do listen to pop.

Write the set that represents the following statements:

(a) The set of students who listen to at least one of the three types of music listed.
   \[ U \cap (C \cup R \cup P) \]

(b) The set of students who listen to neither country nor rock.
   \[ C^c \cap R^c \]

(c) The set of students who listen to rock or pop but not country.
   \[ (R \cup P) \cap C^c \]
10. Let \( n(U) = 102 \), \( n(A) = 18 \), \( n(B) = 38 \), and \( n(A \cap B) = 14 \). Compute:

(a) \( n(A^c \cap B) \)
\[ \frac{4 + 14}{24} = \frac{18}{24} = \frac{3}{4} \]

(b) \( n(B^c) = 60 + 4 = 64 \)

(c) \( n(A^c \cap B^c) = 60 \)

(d) \( n(A^c \cap A) = n(\emptyset) = 0 \)

(e) How many subsets does \( A \) have?
\[ n(A) = 4 + 14 = 18 \implies 2^{18} \]

(f) How many proper subsets does \( A \) have?
\[ 2^{18} - 1 \quad (\text{to exclude } A) \]

(g) How many nonempty proper subsets does \( A \) have?
\[ 2^{18} - 2 \quad (\text{one for } \emptyset, \text{one for } A) \]
11. Let $A, B,$ and $C$ be sets in a universal set $U$. We are given that $n(U) = 150, n(A) = 60, n(B) = 63, n(C) = 93, n(A \cap B) = 33, n(A \cap C) = 42, n(B \cap C) = 36,$ and $n(A \cap B \cap C^c) = 6$. What is $n((A \cup B \cup C)^c)$? What is $n(A^c \cap B^c \cap C)$?
12. 200 college students were polled about what kind of games (Monopoly, Connect Four, and Guess Who) they played growing up. The results are given below.

- 104 students played Monopoly.
- 89 students played Connect Four. 62 students played Guess Who.
- 37 students played Monopoly and Connect Four. \( M \cap C \)
- 29 students played Monopoly and Guess Who. \( M \cap G \)
- 22 students played Connect Four and Guess Who. \( C \cap G \)
- 12 students played all three games growing up. \( M \cap C \cap G \)

\[
\begin{align*}
42 + 25 + 12 + 10 &= 89 \\
25 + 17 + 12 + 10 &= 62 \\
50 + 17 + 12 + 25 &= 104 \\
200 - (50 + 17 + 12 + 10 + 25 + 23 + 12) &= 21
\end{align*}
\]
(a) How many students played none of these games growing up?

\[21\]

(b) How many students played at least one of these games? \(\cap (M \cup U \cup C)\)

\[200 - 21 = 179\]

(c) How many students played Monopoly and Connect Four, but not Guess Who?

\[25\]

(d) How many students played exactly one of these games growing up?

\[50 + 42 + 23 = 115\]
13. A survey was done of 250 people about whether they have visited the continents of Europe, Africa, or South America. The following information was found.

- 45 people have only been to Europe.
- 20 people have only been to Africa.
- 107 people have been to South America.
- 21 people have been to Europe and South America, but not Africa.
- 95 people have been to exactly 1 of these three continents.
- 67 people have been to Africa or South America, but not Europe.
- 66 people have been to both Europe and Africa.
(a) How many people have been to Europe or Africa, but not South America?

(b) How many people have been to all three?

(c) How many people have only been to South America?

(d) How many people have been to exactly 2 of the three?