## M220 Practice Exam I

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- 1. Let A, B, and C be sets. Let  $X = A (B \cap C)$  and  $Y = (A \cup B) C$ . Which of the following is true? (no proof needed).
  - (a)  $X \subset Y$
  - (b)  $Y \subset X$
  - (c) both (a) and (b), i.e. X = Y
  - (d) none of these.
- 2. Write truth tables for (a)  $p \lor (\neg q \to p)$  (b)  $\neg (q \lor (\neg p \land q))$  (c)  $\neg p \to q$ . (You may put these on one table to save space).
- 3. Show that  $(p \land q) \rightarrow p$  is a tautology.
- 4. State the contrapositive, converse and negation of: for all  $x, y \in \mathbb{R}$  if x and y are irrational then xy is irrational. Prove the negation.
- 5. Prove that if  $A \subset B$  and  $B \subset C$  then  $A \subset C$  (for sets A, B and C).
- 6. Let for each real number y > 0, let  $A_y = (-y, y)$  i.e. the set of real numbers x with -y < x < y. Compute  $\bigcap_{y>0} A_y$  and  $\bigcup_{y>0} A_y$ .
- 7. Suppose that A is a set with |A| = n and  $A_1, \ldots, A_m \subset A$  are disjoint subsets such that  $A = \bigcup_{i=1}^m A_i$  and m < n. What does the Pigenhole Principle allow you to conclude?
- 8. For each rational number x with x > 0 define  $A_x = (\sqrt{2} x, x + \sqrt{2})$  (the open interval on the real line).
  - (a) What is  $\bigcup_{x>0} A_x$ ?
  - (b) What is  $\bigcap_{x>0} A_x$ ?
- 9. Let  $f : \mathbb{R} \to \mathbb{R}$  be defined by

$$f(x) := \begin{cases} \frac{x-2}{3x-1} & x \neq 1/3\\ 1/3 & x = 1/3 \end{cases}.$$

Show that f is bijective.