# M220 Practice Exam I 

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1. Let $A, B$, and $C$ be sets. Let $X=A-(B \cap C)$ and $Y=(A \cup B)-C$. Which of the following is true? (no proof needed).
(a) $X \subset Y$
(b) $Y \subset X$
(c) both (a) and (b), i.e. $X=Y$
(d) none of these.
2. Write truth tables for (a) $p \vee(\neg q \rightarrow p)(\mathrm{b}) \neg(q \vee(\neg p \wedge q))(\mathrm{c}) \neg p \rightarrow q$. (You may put these on one table to save space).
3. Show that $(p \wedge q) \rightarrow p$ is a tautology.
4. State the contrapositive, converse and negation of: for all $x, y \in \mathbb{R}$ if $x$ and $y$ are irrational then $x y$ is irrational. Prove the negation.
5. Prove that if $A \subset B$ and $B \subset C$ then $A \subset C$ (for sets $A, B$ and $C)$.
6. Let for each real number $y>0$, let $A_{y}=(-y, y)$ i.e. the set of real numbers $x$ with $-y<x<y$. Compute $\cap_{y>0} A_{y}$ and $\cup_{y>0} A_{y}$.
7. Suppose that $A$ is a set with $|A|=n$ and $A_{1}, \ldots, A_{m} \subset A$ are disjoint subsets such that $A=\cup_{i=1}^{m} A_{i}$ and $m<n$. What does the Pigenhole Principle allow you to conclude?
8. For each rational number $x$ with $x>0$ define $A_{x}=(\sqrt{2}-x, x+\sqrt{2})$ (the open interval on the real line).
(a) What is $\bigcup_{x>0} A_{x}$ ?
(b) What is $\bigcap_{x>0} A_{x}$ ?
9. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$
f(x):=\left\{\begin{array}{ll}
\frac{x-2}{3 x-1} & x \neq 1 / 3 \\
1 / 3 & x=1 / 3
\end{array} .\right.
$$

Show that $f$ is bijective.

