## M220 Practice Exam I

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1. Let $A, B$, and $C$ be sets. Let $X=A-(B \cap C)$ and $Y=(A \cup B)-C$. Which of the following is true? (no proof needed).
(a) $X \subset Y$
(b) $Y \subset X$
(c) both (a) and (b), i.e. $X=Y$
(d) none of these.
2. Write truth tables for (a) $p \vee(\neg q \rightarrow p)(\mathrm{b}) \neg(q \vee(\neg p \wedge q))(\mathrm{c}) \neg p \rightarrow q$. (You may put these on one table to save space).
3. Show that $(p \wedge q) \rightarrow p$ is a tautology.
4. State the contrapositive, converse and negation of: for all $x, y \in \mathbb{R}$ if $x$ and $y$ are irrational then $x y$ is irrational. Prove the negation.
5. Prove that if $A \subset B$ and $B \subset C$ then $A \subset C$ (for sets $A, B$ and $C$ ).
6. Let for each real number $y>0$, let $A_{y}=(-y, y)$ i.e. the set of real numbers $x$ with $-y<x<y$. Compute $\cap_{y>0} A_{y}$ and $\cup_{y>0} A_{y}$.
7. Suppose $A_{1}, \ldots, A_{n}$ are finite disjoint sets and $\left|\cup_{i=1}^{n} A_{i}\right|=5 n+1$. What does the Pigeonhole Principle allow you to conclude?
8. Let $A$ and $B$ be any non-empty sets. Show that if $A \cap B=\varnothing$ then $\{A, B\}$ is a partition of $A \cup B$.
