## M220 Practice Exam I

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- 1. Let A, B, and C be sets. Let  $X = A (B \cap C)$  and  $Y = (A \cup B) C$ . Which of the following is true? (no proof needed).
  - (a)  $X \subset Y$
  - (b)  $Y \subset X$
  - (c) both (a) and (b), i.e. X = Y
  - (d) none of these.
- 2. Write truth tables for (a)  $p \lor (\neg q \to p)$  (b)  $\neg (q \lor (\neg p \land q))$  (c)  $\neg p \to q$ . (You may put these on one table to save space).
- 3. Show that  $(p \land q) \rightarrow p$  is a tautology.
- 4. State the contrapositive, converse and negation of: for all  $x, y \in \mathbb{R}$  if x and y are irrational then xy is irrational. Prove the negation.
- 5. Prove that if  $A \subset B$  and  $B \subset C$  then  $A \subset C$  (for sets A, B and C).
- 6. Let for each real number y > 0, let  $A_y = (-y, y)$  i.e. the set of real numbers x with -y < x < y. Compute  $\bigcap_{y>0} A_y$  and  $\bigcup_{y>0} A_y$ .
- 7. Suppose  $A_1, \ldots, A_n$  are finite disjoint sets and  $|\bigcup_{i=1}^n A_i| = 5n+1$ . What does the Pigeonhole Principle allow you to conclude?
- 8. Let A and B be any non-empty sets. Show that if  $A \cap B = \emptyset$  then  $\{A, B\}$  is a partition of  $A \cup B$ .