1. (a) Define a binary operation on $\mathbb{R}_+$ by $a \ast b = \sqrt{a + b}$. Is this operation associative?

(b) For $a, b \in \mathbb{R}_+$ set $a \ast b = \int_0^1 bxe^{ax}dx$. Is $\ast$ a well-defined binary operation?

2. Let $H = \left\{ \frac{a}{n} : a \in \mathbb{Z}, n \in \mathbb{N} \right\}$. Show that $H < \mathbb{Q}$ under $+$.

3. Let $A := \left\{ \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \in M_2(\mathbb{R}) : ab \neq 0 \right\}$.

   (a) Is $A < GL_2(\mathbb{R})$ (under multiplication)?

   (b) Is $A < M_2(\mathbb{R})$ (under addition)?

4. Let $D := \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \in M_2(\mathbb{R}) : ab \neq 0 \right\}$. Show that $D < GL_2(\mathbb{R})$. Is $D$ abelian?

5. Let $H, K < G$ be subgroups. Show that $H \cap K < G$.

6. Find all solutions $z \in \mathbb{C}$ to $z^5 = -1$. Write your answers in the form $e^{\theta i}$ for $\theta \in \mathbb{R}$.

7. Let $a = e^{2\pi i/36} \in U_{36}$.

   (a) What is $|a^9|$?

   (b) What is $|\langle a^6, a^4 \rangle|$?

8. Define what it means for $\varphi : G \rightarrow H$ to be a group isomorphism.

9. Define $\varphi : \mathbb{Z}_6 \rightarrow U_6$ by $\varphi(\pi) = e^{-2\pi mi/6}$. Show that $\varphi$ is an isomorphism.

10. Determine the order of $a = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \in GL_2(\mathbb{R})$ (a group under multiplication).

11. Show that any finite non-empty subset $H \subset G$ of a group $G$ that is closed under the binary operation is a subgroup. Give an example of a group $G$ and a subset $X \subset G$ that is closed under the binary operation in $G$ but $X$ is not a subgroup.