13.4: Polar Coordinates

The **polar coordinate system** consists of:

- the **pole** (or origin) labeled O;

- the **polar axis** which is a ray starting at O (usually drawn horizontally to the right);

The **polar coordinates** (r, θ) of a point P:

- θ is the angle between the polar axis and the line OP (the angle is positive if measured in counterclockwise direction from the polar axis);

- r is the distance from O to P.

EXAMPLE 1. *Plot the points whose polar coordinates are given:*
   (a) $(1, \pi/3)$ **(b)** $(5, -\pi/2)$.

The connection between polar and Cartesian coordinates:

$$\cos \theta = \qquad\qquad\qquad\qquad \sin \theta =$$

$$x = \qquad\qquad\qquad\qquad y =$$

$$r^2 = \qquad\qquad\qquad\qquad \tan \theta =$$

REMARK 2. In converting from the Cartesian to polar coordinates we must choose θ so that the point (r, θ) lies in the correct quadrant.

EXAMPLE 3. *Convert the point $(4, \pi/6)$ from polar to Cartesian coordinates.*

EXAMPLE 4. *Represent the point with Cartesian coordinates $(-10, 10)$ in terms of polar coordinates.*

EXAMPLE 5. *Find the distance between the points $A(2, \pi/6)$ and $B(3, \pi/3)$ in polar coordinates.*

EXAMPLE 6. *What curve is represented by the polar equation $r = 12$?*

EXAMPLE 7. *What curve is represented by the polar equation $\theta = \pi/4$?*

EXAMPLE 8. *Sketch the region in the Cartesian plane consisting of points whose polar coordinates satisfy the following conditions:* $1 \leq r \leq 2, \quad 0 \leq \theta \leq \pi.$

EXAMPLE 9. *Sketch the curve with polar equation $r = 2\sin\theta$.*

EXAMPLE 10. *Sketch the curve with polar equation $r = 2\cos\theta$.*

EXAMPLE 11. *Sketch the curve* $r = 1 + \cos\theta$.

EXAMPLE 12. *Sketch the curve* $r^2 = 4\cos 2\theta$.