10.5: Power Series

DEFINITION 1. *A power series about $x = a$ (or centered at $x = a$), or just* **power series**, *is any series that can be written in the form*

$$\sum_{n=0}^{\infty} c_n (x-a)^n,$$

where a and c_n are numbers. The c_n's are called the **coefficients** *of the power series.*

THEOREM 2. *For a given power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ there are only 3 possibilities:*

1. *The series converges only for $x = a$.*
2. *The series converges for all x.*
3. *There is $R > 0$ such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$. We call such R the* **radius of convergence.**

REMARK 3. In case 1 of the theorem we say that $R = 0$ and in case 2 we say that $R = \infty$

DEFINITION 4. *An* **interval of convergence** *is the interval of all x's for which the power series converges.*

EXAMPLE 5. *Assume that it is known that the series $\sum_{n=0}^{\infty} c_n (x+3)^n$ converges when $x = -10$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series:*

$\sum_{n=0}^{\infty} c_n 2^n$	$\sum_{n=0}^{\infty} c_n (-11)^n$	$\sum_{n=0}^{\infty} c_n 8^n$

EXAMPLE 6. *Given* $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$.

(a) *Find the radius of convergence.*

(b) *Find the interval of convergence.*

EXAMPLE 7. *Given* $\sum_{n=1}^{\infty} \frac{2^n}{n}(3x-6)^n$.

(a) *Find the radius of convergence.*

(b) *Find the interval of convergence.*

EXAMPLE 8. *Given* $\sum_{n=1}^{\infty} \frac{(-1)^n}{(3n+1)!}(x+8)^n$.

(a) *Find the radius of convergence.*

(b) *Find the interval of convergence.*

EXAMPLE 9. *Given* $\sum_{n=1}^{\infty} \frac{(2n)!}{9^{n-1}}(x+8)^n$.

(a) *Find the radius of convergence.*

(b) *Find the interval of convergence.*

EXAMPLE 10. *Given* $\displaystyle\sum_{n=1}^{\infty} \frac{x^{5n+5}}{3^{n+1}(n+1)}$.

(a) *Find the radius of convergence.*

(b) *Find the interval of convergence.*