

## 6.5: The substitution rule

**The Substitution Rule for indefinite integrals:** If  $u = g(x)$  is a differentiable function, then

$$\int f(g(x))g'(x) dx = \int f(u) du$$

Note that  $du = g'(x)dx$  is differential.

The correct substitution depends on the integral:

$$\begin{array}{lll} \int (g(x))^n g'(x) dx & \int e^{g(x)} g'(x) dx & \int \frac{g'(x)}{g(x)} dx \\ \int \cos(g(x))g'(x) dx & \int \sin(g(x))g'(x) dx & \int \sec^2(g(x))g'(x) dx \\ \int \sec(g(x)) \tan(g(x))g'(x) dx & \int \csc^2(g(x))g'(x) dx & \int \csc(g(x)) \cot(g(x))g'(x) dx \end{array}$$

**EXAMPLE 1.** Evaluate each of the following integrals:

1.  $\int x(x^2 + 2012)^{2012} dx$

2.  $\int 18x^2 \sqrt[4]{6x^3 + 5} dx$

3.  $\int \cos(3x) \sin^{10}(3x) dx$

4.  $\int (8x - 1)e^{4x^2 - x} dx$

$$5. \int \sec^2(5y)(5 - \tan(5y))^5 dy$$

$$6. \int \frac{x}{7x^2 + 12} dx$$

$$7. \int \tan x dx$$

**The Substitution Rule for definite integrals:** If  $u = g(x)$  is a differentiable function, then

$$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

**EXAMPLE 2.** Evaluate each of the following integrals:

$$1. \int_{e^2}^{e^6} \frac{(\ln x)^4}{x} dx$$

2.  $\int_{-1}^1 \frac{1}{(1+3x)^3} - \frac{3}{1+3x} dx$

3.  $\int_0^{0.5} (\sin(\pi y) - e^y) dy$