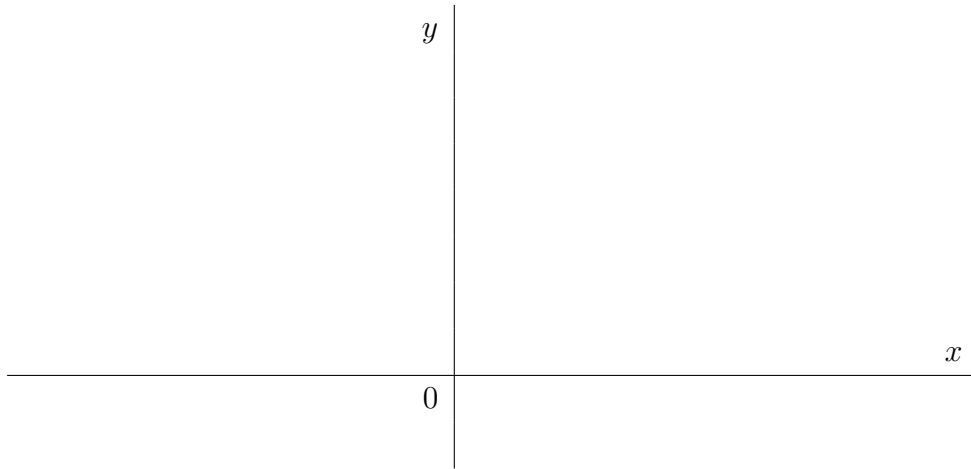


7.3: Volumes by Cylindrical Shells

EXAMPLE 1. Determine the volume of the solid obtained by rotating the region bounded by $y = 4(x - 1)(x - 5)^2$ and the x -axis about the y -axis.



Let us list the troubles in application washer method here:

$$V =$$

How to avoid these troubles?

Unlike the washer method in shell method the cross sections are always **parallel** to the axis of revolution.

Think of cutting your solid by a cylindrical cutter centered on the axis of revolution and pushed down into the solid.

A cylindrical shell is a solid bounded by two concentric circular cylinders with the same height. The volume of the cylindrical shell:

Solution of Example 1:

SUMMARY (Method of Cylindrical Shells)

- Area of cross sections: $A(x) = 2\pi(\text{radius})(\text{height})$

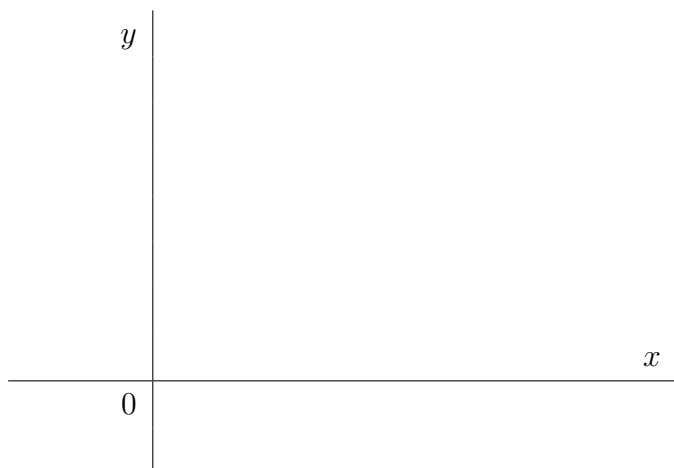
- For rotation about a *vertical* axis we use $V = \int_a^b A(x) dx$.

For rotation about a *horizontal* axis we use $V = \int_c^d A(y) dy$

Note: Exactly opposite of washer method.

- For the limits of integration we take the only range of x or y covering one side of the solid (not the complete range).

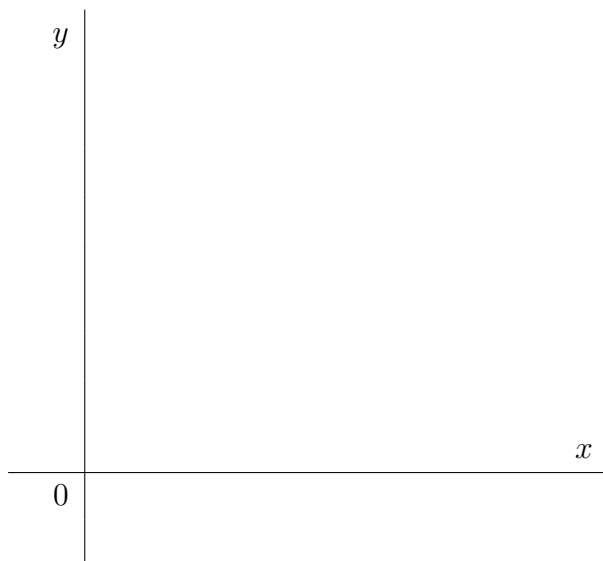
EXAMPLE 2. Find the volume of the solid obtained by rotating the region enclosed by the curves $y = x - x^2$ and $y = 0$ about the line $x = 2$.



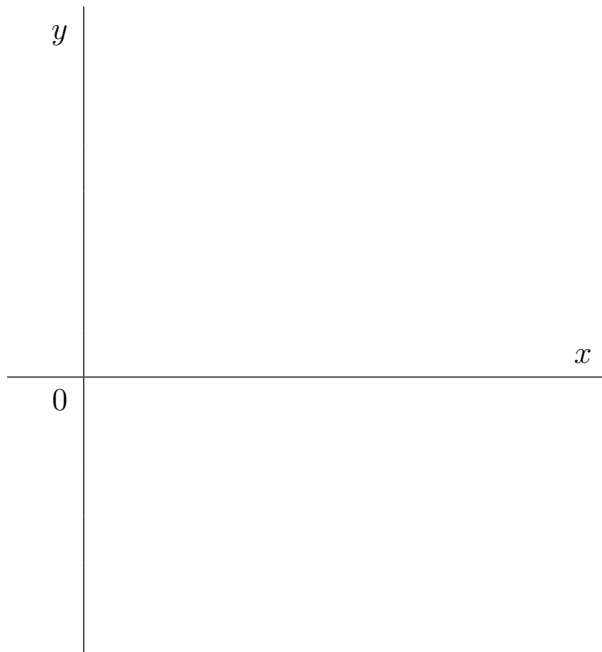
EXAMPLE 3. Set up, but do not evaluate, the volume of the solid obtained by rotating the region enclosed by the curves $y = \sqrt[3]{x}$, $x = 8$, $y = 0$ about the x -axis using the method of cylindrical shells and then using the washer method.



EXAMPLE 4. Set up, but do not evaluate, the volume of the solid obtained by rotating the region enclosed by the curves $y = \cos x$, $x = 0$, $x = \pi/2$ and $y = 0$ about the line $y = 1$ using the method of cylindrical shells and then using the washer method.



EXAMPLE 5. Set up, but do not evaluate, the volume of the solid obtained by rotating the region enclosed by the curves $y = x^2$ and $y = 4x$ about the x -axis using the method of cylindrical shells and then using the washer method.



EXAMPLE 6. Solve the last problem from Section 7.2 using shells:
Find the formula for volume of a solid torus with radii r and R .