

### 8.3: Trigonometric Substitution

- $\int x\sqrt{x^2 - 4} \, dx$

- $\int \frac{x}{\sqrt{x^2 - 4}} \, dx$

EXAMPLE 1. Evaluate  $I = \int \frac{\sqrt{x^2 - 4}}{x} \, dx$

EXAMPLE 2. Evaluate  $I = \int_2^4 \frac{\sqrt{x^2 - 4}}{x} dx$

Table of Inverse Trigonometric Substitutions

$\sqrt{a^2 - b^2x^2}$	$x = \frac{a}{b} \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \sin^2 \theta = \cos^2 \theta$
$\sqrt{a^2 + b^2x^2}$	$x = \frac{a}{b} \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2 \theta = \sec^2 \theta$
$\sqrt{b^2x^2 - a^2}$	$x = \frac{a}{b} \sec \theta, \quad 0 \leq \theta < \frac{\pi}{2} \quad \text{or} \quad \pi \leq \theta < \frac{3\pi}{2}$	$\sec^2 \theta - 1 = \tan^2 \theta$

EXAMPLE 3. Evaluate  $I = \int \frac{1}{x^4 \sqrt{4 - 25x^2}} dx$

EXAMPLE 4. Evaluate  $I = \int \frac{x^5}{(x^2 + 36)^{3/2}} dx$

EXAMPLE 5. Evaluate  $I = \int \frac{x}{\sqrt{2x^2 - 4x - 7}} dx$

EXAMPLE 6. Evaluate  $I = \int \frac{x}{\sqrt{3 - 2x - x^2}} dx$