

9.4: Area of a Surface of Revolution

Question: *What is the area of the band (or frustum of a cone?)*

Answer:

PROBLEM: Find the surface area of solid obtained by rotating the curve

$$y = f(x), a \leq x \leq b$$

about the x -axis. (Assume that f is nonnegative and continuous on $[a, b]$.)

Solution: Approximate the surface area by areas of approximating bands:



EXAMPLE 1. Find the surface area of the solid obtained by rotating the curve

$$C : x = R \cos t, \quad y = R \sin t, \quad 0 \leq t \leq \pi \quad (R > 0)$$

about the x -axis.

EXAMPLE 2. Find the surface area of the solid obtained by revolving the curve given by

$$y^2 = 4x + 4, \quad 2 \leq y \leq 6,$$

about the x -axis.

EXAMPLE 3. Determine the surface area of the solid obtained by revolving the curve given by

$$y = \sqrt[3]{x}, \quad 1 \leq x \leq 8,$$

about the y -axis.

EXAMPLE 4. Determine the surface area of the solid obtained by rotating

$$y = \sqrt{9 - x^2}, \quad |x| \leq 2,$$

about the x -axis.