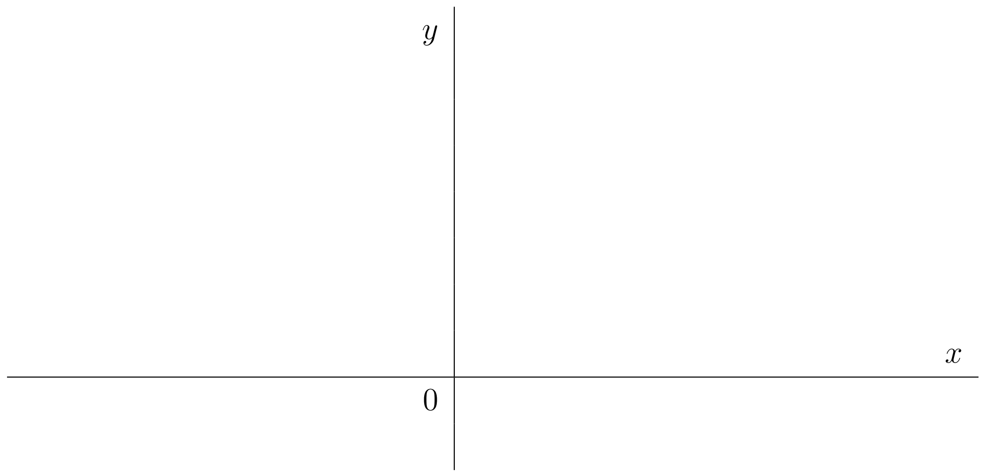
7.3: Volumes by Cylindrical Shells

EXAMPLE 1. *Determine the volume of the solid obtained by rotating the region bounded by* $y = 4(x-1)(x-5)^2$ *and the* x*-axis about the* y*-axis.*



Let us list the troubles in application washer method here:

$$V =$$

How to avoid these troubles?

Unlike the washer method in shell method the cross sections are always **parallel** to the axis of revolution.

`Think of cutting your solid by a cylindrical cutter centered on the axis of revolution and pushed down into the solid.`

A `cylindrical shell` is a solid bounded by two concentric circular cylinders with the same height. The volume of the cylindrical shell:

Solution of Example 1:

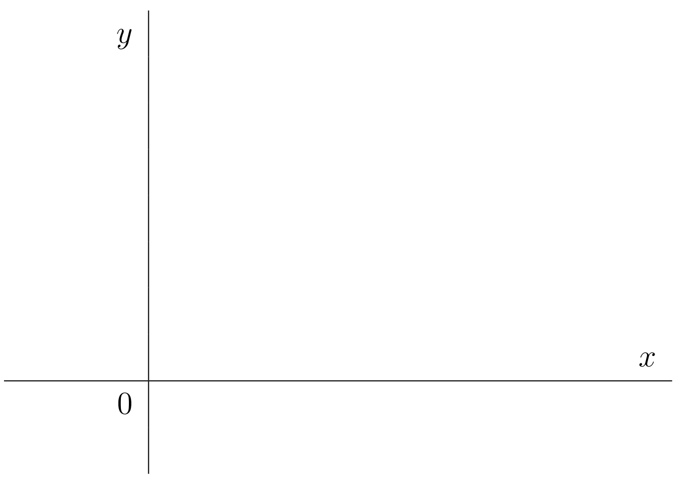
SUMMARY (Method of Cylindrical Shells)

- Area of cross sections: $A(x) = 2\pi(\text{radius})(\text{height})$
- For rotation about a *vertical* axis we use $V = \displaystyle\int_a^b A(x)\,\mathrm{d}x$.

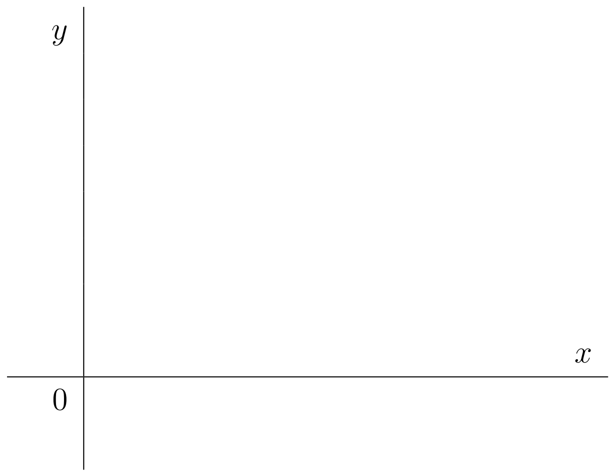
  For rotation about a *horizontal* axis we use $V = \displaystyle\int_c^d A(y)\,\mathrm{d}y$

  Note: Exactly opposite of washer method.
- For the limits of integration we take the only range of x or y covering one side of the solid (not the complete range).

EXAMPLE 2. *Find the volume of the solid obtained by rotating the region enclosed by the curves $y = x - x^2$ and $y = 0$ about the line $x = 2$.*



EXAMPLE 3. *Set up, but do not evaluate, the volume of the solid obtained by rotating the region enclosed by the curves* $y = \sqrt[3]{x}, x = 8, y = 0$ *about the* x*-axis using the method of cylindrical shells and then using the washer method.*



EXAMPLE 4. *Set up, but do not evaluate, the volume of the solid obtained by rotating the region enclosed by the curves* $y = \cos x, x = 0, x = \pi/2$ *and* $y = 0$ *about the line* $y = 1$ *using the method of cylindrical shells and then using the washer method.*