7.5: Average Value of a Function

The average value of finitely many numbers $y_1, y_2, \ldots, y_n$:

The **average value of a function** $y = f(x)$ over the interval $[a, b]$:

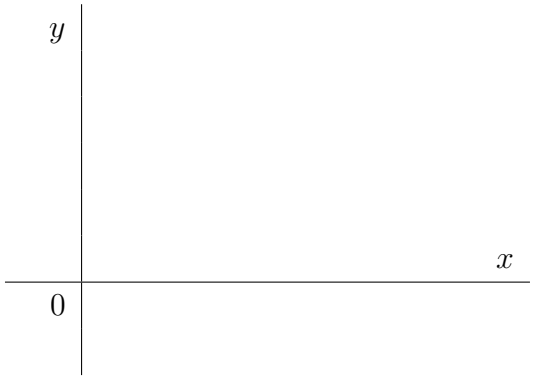
$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x.$$

EXAMPLE 1. *Determine the average value of* $f(x) = x^2 - 4x + 7\sin(\pi x)$ *over the interval* $[-\frac{1}{2}; \frac{1}{2}]$.

EXAMPLE 2. *The temperature of a metal rod, 10 m long, is* $5x$ *(in* $^\circ C$*) at a distance* x *meters from one end of the rod. What is the average temperature of the rod?*

MEAN VALUE THEOREM FOR INTEGRALS: *If f is continuous on $[a, b]$, then there exists a number c on $[a, b]$ s.t.*

$$\int_a^b f(x)\,\mathrm{d}x = f(c)(b-a).$$



The geometric interpretation of the Mean Value Theorem for Integrals: for *positive* functions f, there is a number c s.t. the rectangle with base $[a, b]$ and height $f(c)$ has the same area as the region under the graph of f from a to b.

Proof.

EXAMPLE 3. *If g is continuous and $\int_{-1}^{7} g(x)\,\mathrm{d}x = 24$ show that g takes on the value 3 at least once on the interval $[-1, 7]$.*

EXAMPLE 4. *Determine the number c that satisfies the Mean Value Theorem for Integrals for the function $f(x) = x^2 - 2x - 2$ on the interval [1,4]*