8.2: Trigonometric Integrals

RULE: How to evaluate $\int \sin(Ax)\cos(Bx)\,\mathrm{d}x$, $\int \sin(Ax)\sin(Bx)\,\mathrm{d}x$, $\int \cos(Ax)\cos(Bx)\,\mathrm{d}x$ *Use the following identities:*

$$\begin{aligned} \sin A\cos B &= \tfrac{1}{2}\left(\sin(A-B)+\sin(A+B)\right) \\ \sin A\sin B &= \tfrac{1}{2}\left(\cos(A-B)-\cos(A+B)\right) \\ \cos A\cos B &= \tfrac{1}{2}\left(\cos(A-B)+\cos(A+B)\right) \end{aligned}$$

EXAMPLE 1. *Evaluate* $I = \int \cos(25x)\cos(4x)\,\mathrm{d}x$

EXAMPLE 2. *Prove th formula, where* m *and* n *are positive integers:*

$$I(m,n) = \int_{-\pi}^{\pi} \sin mx \sin nx\,\mathrm{d}x = \begin{cases} 0 & \text{if} \quad m \neq n \\ \pi & \text{if} \quad m = n \end{cases}$$

$$\begin{array}{ccccc} \sin^2 x & + & \cos^2 x & = & 1 \\ \tan^2 x & + & 1 & = & \sec^2 x \end{array}$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin x \cos x = \frac{1}{2}\sin 2x$$

EXAMPLE 3. *Evaluate* $I = \int \cos x \sin^{2016} x \, \mathrm{d}x$

EXAMPLE 4. *Evaluate* $I_1 = \int_0^{\pi/2} \sin^2 x \, \mathrm{d}x$ *and* $I_2 = \int_0^{\pi/2} \cos^2 x \, \mathrm{d}x$

EXAMPLE 5. *Evaluate* $I = \int \sin^5 x \, \mathrm{d}x$