8.3: Trigonometric Substitution

integral with	substitution	identity
$a^2 - x^2$	$x = a\sin\theta, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$a^2 + x^2$	$x = a\tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$x^2 - a^2$	$x = a\sec\theta, \quad 0 \le \theta < \frac{\pi}{2} \quad \text{or} \quad \pi \le \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$
$ax^2 + bx + c$	complete squares and then do the correct substitution	

EXAMPLE 1. *Use a trigonometric substitution to eliminate the radical:*

(a) $\sqrt{1 - x^2}$

(b) $\sqrt{1 - 3x^2}$

(c) $\sqrt{x^2 + 16}$

(d) $\sqrt{49x^2 + 16}$

EXAMPLE 2. *Evaluate the given integral:*

(a) $\displaystyle\int \frac{1}{x^2\sqrt{5-x^2}}\,\mathrm{d}x$

(b) $\displaystyle\int_{\sqrt{5}/2}^{\sqrt{10}/2} \frac{1}{x^2\sqrt{5-x^2}}\,\mathrm{d}x$

(c) $\displaystyle\int_0^{4/7} \frac{1}{(49x^2+16)^{3/2}}\,\mathrm{d}x$