9.2 First-Order Linear Differential Equations

A first order ODE is called **linear** if it is expressible in the form

$$y' + p(t)y = g(t) \tag{1}$$

where $p(t)$ and $g(t)$ are given functions.
The method to solve (1) for arbitrary $p(t)$ and $q(t)$ is called

The Method of Integrating Factors

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Step 1 Put ODE in the form (1).

Step 2 Find the integrating factor

$$\mu(t) = e^{\int p(t)\mathrm{d}t}$$

Note: Any μ will suffice here, thus take the constant of integration $C = 0$.

Step 3 Multiply both sides of (1) by μ and use the Product Rule for the left side to express the result as

$$(\mu(t)y(t))' = \mu g(x) \tag{2}$$

Step 4 Integrate both sides of (2). Note: Be sure to include the constant of integration in this step!

Step 5 Solve for the solution $y(t)$.

EXAMPLE 1. *Solve the following DE:* $y' - 3xy = -xe^{x^2}$.

EXAMPLE 2. *Consider* $y' - 2y = \cos(3t)$.

1. *Find the general solution.*

2. *Find the solution satisfying the initial condition* $y(0) = -2/13$.