6.6: The Logarithm Defined As An Integral

DEFINITION 1. *The* **natural logarithmic function** *is the function defined by*

$$\ln x = \int_1^x \frac{1}{t} \, dt, \quad x > 0. \tag{1}$$

EXAMPLE 2. *Using a geometric interpretation of the definite integral show that*

1. $\ln x > 0$ *if* $x > 1$

2. $\ln x < 0$ *if* $0 < x < 1$

3. $\ln x = 0$ *if* $x = 1$

Applying the Fundamental Theorem of Calculus (Part I) to (1), we get

Laws of Logarithms

If x and y are positive numbers and r is a rational number, then

1. $\ln(xy) = \ln x + \ln y$

2. $\ln \dfrac{x}{y} = \ln x - \ln y$

3. $\ln(x^r) = r \ln x$

Proof.