8.1: Integration by Parts

- $\int x^n e^{kx}\,\mathrm{d}x$
- $\int x^n \sin(kx)\,\mathrm{d}x$, $\int x^n \cos(kx)\,\mathrm{d}x$
- $\int x^n \ln x\,\mathrm{d}x$
- $\int e^x \cos x\,\mathrm{d}x$, $\int e^x \sin x\,\mathrm{d}x$
- $\int \sec^n x\,\mathrm{d}x$ and some integrals involving inverse trigonometric functions.

The **integration by parts formula**:

$$\int fg'\,\mathrm{d}x = fg - \int f'g\,\mathrm{d}x$$

Proof:

Rewrite the above formula using the following substitutions:

$$u = f(x), \qquad v = g(x)$$

LIPET rule to choose u and dTEPIL rule to choose $\mathrm{d}v$:

whichever function comes first in the list below, it should be u.

L Logarithmic functions

I Inverse trigonometric functions

P Polynomial functions

E Exponential functions

T Trigonometric functions

EXAMPLE 1. *Evaluate* $I = \int x\cos(5x)\,\mathrm{d}x$

Integration by parts formula for definite integrals:

$$\int_a^b u\,\mathrm{d}v = uv|_a^b - \int_a^b v\,\mathrm{d}u,$$

where

$$uv|_a^b = u(b)v(b) - u(a)v(a).$$

EXAMPLE 2. *Evaluate* $I = \int_{-1}^{3} xe^{3x}\,\mathrm{d}x$

EXAMPLE 3. *Evaluate* $I = \int x^2 \sin(5x)\,\mathrm{d}x$

EXAMPLE 4. *Evaluate* $I = \int \ln x\,\mathrm{d}x$

EXAMPLE 5. *Evaluate* $I = \int \arcsin x\,\mathrm{d}x$

EXAMPLE 6. *Evaluate* $I = \int e^x \cos x \, \mathrm{d}x$