13.4: Polar Coordinates

REVIEW:

The connection between polar and Cartesian coordinates:

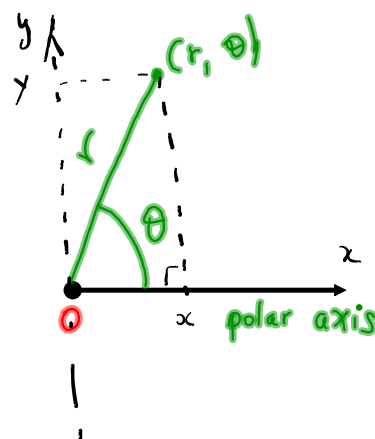$$\cos\theta = \frac{x}{r} \qquad\qquad \sin\theta = \frac{y}{r}$$

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$

$$r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2+y^2} \qquad \tan\theta = \frac{y}{x}$$

REMARK 1. In converting from the Cartesian to polar coordinates we must choose θ so that the point (r,θ) lies in the correct quadrant.

$$r \geq 0 \qquad 0 \leq \theta \leq 2\pi$$

EXAMPLE 2. *What curve is represented by the polar equation*

(a) $r = 12$

$$r = \sqrt{x^2 + y^2} = 12 \implies x^2 + y^2 = 12^2$$

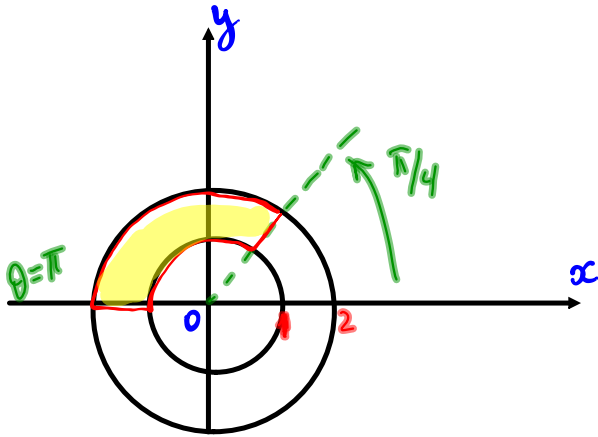circle centered at origin with radius 12
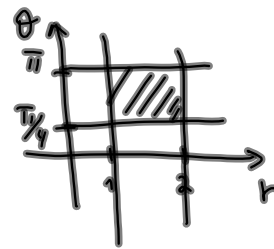
(b) $\theta = \frac{\pi}{3}$

ray which makes angle $\pi/3$ with positive direction of polar axis

EXAMPLE 3. *Sketch the region in the Cartesian plane consisting of points whose polar coordinates satisfy the following conditions:* $1 \leq r \leq 2, \quad \pi/4 \leq \theta \leq \pi.$

$r = 1 \implies x^2 + y^2 = 1$

$r = 2 \implies x^2 + y^2 = 2^2$

compare with

EXAMPLE 4. *Find a polar equation for the curve represented by the given Cartesian equation:*
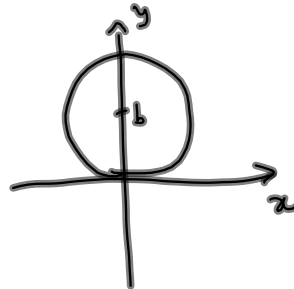
(a) $x^2 + y^2 = 2by$

$x = r\cos\theta$, $y = r\sin\theta$ $\Bigg\}$ $r^2 = 2br\sin\theta$

$$\boxed{r = 2b\sin\theta}$$

Note that this is circle

$$x^2 + y^2 - 2by + b^2 - b^2 = 0$$

$$\underbrace{}_{(y-b)^2}$$

$$x^2 + (y-b)^2 = b^2$$

(b) $(x-a)^2 + y^2 = a^2$

$$x^2 - 2ax + a^2 + y^2 = a^2$$

$$x^2 + y^2 = 2ax \quad \Rightarrow \quad r^2 = 2ar\cos\theta \Rightarrow \boxed{r = 2a\cos\theta}$$