Math 131 Week in Review
Sections 1.1-1.3
9/6/09

1. The graph of a function \( f \) is given.

(a) Estimate the value of \( f(2) \).
(b) Estimate the value of \( f(-1) \).

(c) For what value(s) of \( x \) is \( f(x) = 1 \)?
(d) Estimate the value of \( x \) such that \( f(x) = 0 \).

(e) Give the domain and range of \( f \).
   \[ D: (-2, 4) \quad R: (-1, 3) \]

(f) On what interval(s) is \( f \) increasing?
   \[ (-2, 1) \quad (3, 4) \]
2. Determine whether the curve is a graph of a function of \( x \). State the domain and range.

- **Yes, passes the vertical line test**

  \[ D: (-2, 3.9] \]
  \[ R: (-1, 4] \]

3. Evaluate the difference quotient for \( f(x) = \frac{2}{x+1} \):

\[
\frac{f(a+h) - f(a)}{h} = \frac{\frac{2}{a+h+1} - \frac{2}{a+1}}{h}
\]

\[
= \frac{2(a+1) - 2(a+h+1)}{(a+1)(a+h+1)} \cdot \frac{1}{h}
\]

\[
= \frac{2a+2 - (2a+2h+2)}{(a+1)(a+h+1)} \cdot \frac{1}{h}
\]

\[
= \frac{2a+2 - 2a - 2h}{(a+1)(a+h+1)h}
\]

\[
= \frac{-2h}{(a+1)(a+h+1)h}
\]

\[
= \frac{-2}{(a+1)(a+h+1)}
\]
4. Find the domain of \( g(x) = \frac{\sqrt{x-2}}{x^2 - 5x + 6} \).
   \[(x-2)(x-3) \neq 0 \]
   \[x \neq 2 \quad x \neq 3 \]

5. Find the domain of \( h(x) = \frac{x - 3}{\sqrt{x^3 - 4x}} \).
   \[x^3 - 4x > 0 \]
   \[x(x^2 - 4) > 0 \]
   \[x(x+2)(x-2) > 0 \]
   \[\text{pos} \]

6. Sketch the graph of \( F(x) = \begin{cases} 
\frac{x^2 - 1}{x + 2} & \text{if } x < 1 \\
\frac{x^2 - 1}{x + 2} & \text{if } x \geq 1 
\end{cases} \)
7. A rectangle has area $25 \text{ m}^2$. Express the perimeter of the rectangle as a function of the length of one of its sides.

\[ A = lw = 25 \]
\[ w = \frac{25}{l} \]

\[ p(l) = 2l \left( \frac{25}{l} \right) + 2l \]
\[ p(l) = \frac{50}{l} + 2l \] or \[ p(l) = \frac{50 + 2l^2}{l} \]

8. Determine whether $f$ is even, odd, or neither: \( f(x) = 1 - 3x^3 - x^5 \)

\[ f(-x) = f(x) \]
\[ f(-x) = -f(x) \]

\[ f(-x) = 1 - 3(-x)^3 - (-x)^5 \]
\[ = 1 + 3x^3 + x^5 \]
\[ \neq f(x) \]
\[ \neq -f(x) \]

neither
9. Classify each of the functions below as a power function, root function, polynomial function, rational function, algebraic function, trigonometric function, exponential function, and/or logarithmic function.

\[ G(x) = x^2 \quad \text{exponential} \quad f(x) = \cos x \cot x \quad \text{trigonometric} \]

\[ h(x) = \frac{3x^4 - 1}{1 - x} \quad \text{rational} \]

\[ F(x) = 2.3x^3 - 2x^2 + \sqrt{x} \quad \text{algebraic} \]

\[ g(x) = x^3 \quad \text{power} \]

\[ H(x) = 3x^{13} \quad \text{power, root} \]

10. A study of deer ticks found a relationship between the density of acorns produced in the fall and the density of deer tick larvae the following spring. With 10 acorns per square meter in the fall, there were 570 ticks larvae per 400 square meters the following spring. With 15 acorns, there were 740 tick larvae per 400 square meters.

(adapted from Lial, Greenwell, & Ritchey, Finite Mathematics and Calculus with Applications, 2002, p. 18)

(a) Find a linear equation that models the number of tick larvae, \( T \), per 400 square meters as a function of the number of acorns, \( A \), per square meter the previous fall.

\[
\begin{align*}
(10, 570) & \quad (15, 740) \\
\frac{y - 570}{15 - 10} & = \frac{740 - 570}{5} \\
5y - 2850 & = 340 \\
y & = 34x + 230 \\
T(A) & = 34A + 230
\end{align*}
\]

(b) What is the slope of the graph? What does it represent?

\[ m = 34 \]

For each increase of 1 acorn per m² in the fall, we expect 34 more tick larvae per 400 m² the following spring.

(c) If 30 acorns per square meter are found in the fall, what is the expected number of tick larvae the following spring?

\[
T(30) = 34(30) + 230 = 1020 + 230 = 1250 \text{ Tick larvae } / 400 \text{ m}^2
\]
11. The following data show the connection between blood sugar levels and cholesterol levels for 8 different patients.
(adapted from Lial, Greenwell, & Ritchey, *Finite Mathematics and Calculus with Applications*, 2002, p. 46)

<table>
<thead>
<tr>
<th>Patient</th>
<th>Blood Sugar Level (x)</th>
<th>Cholesterol Level (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>130</td>
<td>170</td>
</tr>
<tr>
<td>2</td>
<td>138</td>
<td>160</td>
</tr>
<tr>
<td>3</td>
<td>142</td>
<td>173</td>
</tr>
<tr>
<td>4</td>
<td>159</td>
<td>181</td>
</tr>
<tr>
<td>5</td>
<td>165</td>
<td>201</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>192</td>
</tr>
<tr>
<td>7</td>
<td>210</td>
<td>240</td>
</tr>
<tr>
<td>8</td>
<td>250</td>
<td>290</td>
</tr>
</tbody>
</table>

(a) Make a scatter plot of these data and decide whether a linear model is appropriate. Yes

(b) Find and graph a line model using the first and last data points.

$$m = \frac{290-170}{250-130} = \frac{120}{120} = 1$$

$$y - 170 = 1(x - 130)$$

$$y = x + 40$$

(c) Find and graph the least squares regression line.

$$y = 0.972x + 31.427$$

(d) Use the linear model in part (c) to estimate the cholesterol for a blood sugar level of 150.

$$y(150) \approx 177$$

(e) Use the linear model in part (c) to estimate the blood sugar level for a cholesterol level of 260.

$$x = 235$$
12. The graph of $f$ is given below.

Use the graph of $f$ to graph the following functions.

(a) $y = f(\frac{1}{2}x)$
   - horizontal stretch

(b) $y = 2f(x) + 1$
   - vertical stretch
   - shifts up 1
   - shift right 2

(c) $y = -f(x - 2)$
   - reflection across X-axis
13. Use parent functions and transformations to graph

(a) \( y = x^2 - 4x + 3 \)
   \[ y = x^2 - 4x + 4 + 3 - 4 \]
   \[ y = (x - 2)^2 - 1 \]

(b) \( y = |x + 1| - 2 \)
   \[ \text{left 1} \]
   \[ \text{down 2} \]

(c) \( y = 2\sqrt{x} - 1 \)
   \[ \text{vertical stretch} \ x^2 \]
   \[ \text{down 1} \]
14. For \( f(x) = 3x - 1 \) and \( g(x) = \sqrt{x} + 1 \), find \( f \circ g \) and \( g \circ f \) and their domains:

\[
(f \circ g)(x) = f(g(x)) = f(\sqrt{x} + 1) = 3(\sqrt{x} + 1) - 1 = 3\sqrt{x} + 3 - 1 = 3\sqrt{x} + 2 \quad D: [\frac{1}{3}, \infty)
\]

\[
(g \circ f)(x) = g(f(x)) = g(3x - 1) = \sqrt{3x - 1} + 1 \quad D: [\frac{1}{3}, \infty)
\]

3x - 1 \geq 0
x \geq \frac{1}{3}

15. Express the function in the form \( f \circ g \circ h \)

\[
H(x) = \sqrt[3]{e^x - 2}
\]

\[
h(x) = e^x \\
g(x) = x^2 - 2 \\
f(x) = \sqrt[3]{x}
\]

\[
h(x) = x \Rightarrow f(g(h(x))) = f(\sqrt[3]{e^x}) = f\left(\sqrt[3]{e^x}\right) = \sqrt[3]{e^x - 2}
\]

\[
g(x) = e^x - 2 \Rightarrow f(g(x)) = f(e^x - 2) = \sqrt[3]{e^x - 2}
\]

\[
h(x) = x \Rightarrow f(g(h(x))) = f(\sqrt[3]{x}) = f\left(\sqrt[3]{x}\right) = \sqrt[3]{e^x - 2}
\]