Math 131 Week in Review  
Sections 1.5-1.6, 2.1  
03/10

1. Use the laws of exponents to simplify each of the following:

(a) \( \frac{(6x^2y^3)^2}{\sqrt{x}} \)  
\[ \frac{6^2x^4y^6}{x^{1/2}} = 216x^{7/2}y^5 \]

(b) \( (a^{-1}b^2)^{n+1} \)  
\[ = a^{-n-1}b^{2n+2} \]

2. Explain the relationship between the graphs of the functions below:

- \( y = 3^x \)  
- \( y = 6^x \)  
- \( y = 15^x \)  
- \( y = 2^x \)

As base gets larger, the graph rises faster.

all go through \((0,1)\) and \((1,b)\) where \(b\) is the base.
3. Starting with the graph of \( f(x) = e^x \), write the function \( g \), resulting from reflecting the graph about the line \( y = 2 \) and translating right 3 units. Sketch the graphs of \( f \) and \( g \).
4. Find the domain of each function.

\[ f(x) = \frac{1}{\sqrt{1 - 2^x}} \]

\[ g(x) = \frac{1 + 2x}{1 - x^2} \]

5. The half-life of sodium-24, $^{24}\text{Na}$, is 15 hours.

(a) If a sample has a mass of 30 mg, find the amount remaining after 45 hours.

\[ A(t) = 30 \left(\frac{1}{2}\right)^{t/15} \]

(b) Find the amount remaining after 1 hour.

\[ A(15) = 30 \left(\frac{1}{2}\right)^{t/15} \approx 3.625 \text{ mg} \]

(c) Estimate the amount remaining after 2 days.

\[ A(48) = 30 \left(\frac{1}{2}\right)^{t/15} \]

(d) Estimate the time required for the mass to be reduced to 2 mg.

\[ 2 = 30 \left(\frac{1}{2}\right)^{t/15} \]

\[ \frac{1}{15} = \left(\frac{1}{2}\right)^{t/15} \]

\[ \ln \frac{1}{15} = \ln \left(\frac{1}{2}\right) \]

\[ \ln \frac{1}{15} = \frac{1}{15} \ln (0.5) \]

\[ 15 \ln \frac{1}{15} = t \ln (0.5) \]

\[ t = \frac{15 \ln (0.5)}{\ln (0.5)} \]

\[ t = 58.6 \text{ hours} \]
6. Determine whether the following functions are one-to-one.

(a) \( f(x) = x^2 + 3x - 2 \)

\( f\) is not one-to-one because it does not pass the horizontal line test.

(b) \( g(x) = \frac{1}{x} \)

\( g\) is one-to-one because it passes the horizontal line test twice.

(c) \( h(x) = \sqrt{x} \)

\( h\) is not one-to-one because it does not pass the horizontal line test.
7. Find the inverse of \( g(x) = \frac{2x+1}{3x-4} \)

\[
\begin{align*}
x &= \frac{2y+1}{3y-4} \\
x(3y-4) &= 2y+1 \\
3xy-4x &= 2y+1 \\
3xy-4x &= 2y+1 \\
x &= \frac{1+4x}{3x-2} \\
g^{-1}(x) &= \frac{1+4x}{3x-2}
\end{align*}
\]

8. Find the inverse of \( h(x) = \ln(x-2)+1 \)

\[
\begin{align*}
x &= \ln(y-2)+1 \\
x-1 &= \ln(y-2) \\
\frac{x}{1} &= \frac{y-2}{1} \\
e^x &= y-2 \\
y &= e^{x+2}
\end{align*}
\]

9. Sketch the inverse of the function graphed below.

To place the line \( y=x \) properly, we must assume the \( x \)- and \( y \)-scale are the same.
10. If \( f(x) = x^2 - 3x + 1 \), find \( f^{-1}(1) \) and \( f^{-1}(-1) \).

\( f^{-1} \) is the inverse function.

- \( f^{-1}(1) \) is the \( x \)-value where \( f(x) = 1 \).
- \( f^{-1}(-1) \) is the \( x \)-value where \( f(x) = -1 \).

\[ f'(1) = 2x - 3 \]
\[ f'(2) = 1 \]
\[ f'(3) = 0 \]
\[ f'(4) = 3 \]

Note: the inverse is not a function. It is a relation.

11. The graph of \( f \) is given.

a. What are the domain and range of \( f^{-1} \)?

- Domain: \( (-\infty, \infty) \)
- Range: \( (-3, 4) \)

b. Estimate the value of \( f^{-1}(3) \).

c. Estimate the value of \( f^{-1}(0) \).

Looking for \( x \)-values of \( f \) same as \( y \)-values of \( f^{-1} \).
12. Find the inverse of \( f(x) = \frac{e^x}{1-2e^x} \).

\[
\begin{align*}
x = \frac{e^y}{1-2e^y} \\
x(1-2e^y) = e^y \\
x - 2xe^y = e^y \\
e^y + 2xe^y = x
\end{align*}
\]

\[
\begin{align*}
e^y(1+2x) &= x \\
e^y &= \frac{x}{1+2x} \\
\ln e^y &= \ln \frac{x}{1+2x} \\
y &= \ln \frac{x}{1+2x}
\end{align*}
\]

13. Find the exact value of each of the following:

(a) \(2 \log_6 36\)  
(b) \(\ln \frac{1}{e}\)  
(c) \(\log \sqrt[3]{10}\)

\[
\begin{align*}
2 \log_6 36 &= 2 \log_6 6^2 = 2 \cdot 2 = 4 \\
\ln \frac{1}{e} &= -1 \\
\log \sqrt[3]{10} &= \log 10^{\frac{1}{3}} = \frac{1}{3}
\end{align*}
\]

14. Express as a single logarithm: \(\log_2 \sin x + \log_2 (x+3) - \frac{1}{2} \log_2 5\)

\[
\begin{align*}
\log_2 \left[\frac{(x+3) \sin x}{\sqrt[3]{5}}\right] &- \frac{1}{2} \log_2 5 \quad \text{Law 1} \\
= \log_2 \left[\frac{(x+3) \sin x}{\sqrt[3]{5}}\right] - \log_2 5^\frac{1}{2} \quad \text{Law 3} \\
= \log_2 \left[\frac{(x+3) \sin x}{\sqrt[3]{5}}\right] - \log_2 \sqrt[3]{5} \\
= \log_2 \left(\frac{(x+3) \sin x}{\sqrt[3]{5}}\right) \quad \text{Law 2} \\
= \log_2 \left(\frac{5(x+3) \sin x}{5}\right) \quad \text{simplest radical form}
\end{align*}
\]
15. Solve for x: \[ \frac{\ln x - \ln (x+2)}{x+2} = 1 \]

\[ e^x + 2e = x \]
\[ xe - x = -2e \]
\[ x(e-1) = 2e \]
\[ x = \frac{2e}{e-1} \]

Since the value is \( < 0 \), there is no solution.

16. Solve for x: \[ e^{2x} + 3e^x = 10 \]

Let \( e^x = y \)

\[ y^2 + 3y - 10 = 0 \]
\[ (y+5)(y-2) = 0 \]

\( y = 5 \) or \( y = -2 \)

Since \( y = 5 \),

\[ x = \ln 5 \]

17. Solve for x: \[ e^{2x^2} > 5 \]

\[ \ln e^{2x^2} > \ln 5 \]
\[ 2x^2 > \ln 5 \]
\[ x^2 > \frac{\ln 5}{2} \]
\[ x > \frac{1}{2} \ln 5 + \frac{3}{2} \]

or \[ x > \ln \sqrt{5} + 1.5 \]
18. The table shows the position of a walker.

<table>
<thead>
<tr>
<th>t (seconds)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x (yards)</td>
<td>0</td>
<td>2.5</td>
<td>6.5</td>
<td>12</td>
<td>15.5</td>
<td></td>
</tr>
</tbody>
</table>

Find the average velocity for each time period.

(a) \([1, 3]\) 

\[
\text{Average velocity} = \frac{9.5 - 2.5}{3-1} = \frac{7}{2} = 3.5 \text{ yards/sec}
\]

(b) \([2, 3]\) 

\[
\text{Average velocity} = \frac{9.5 - 6.5}{3-2} = 3 \text{ yards/sec}
\]

(c) \([3, 4]\) 

\[
\text{Average velocity} = \frac{12 - 9.5}{4-3} = 2.5 \text{ yards/sec}
\]