Math 131 Week in Review
Sections 1.1-1.3, 1.5-1.6, 2.1-2.6
2/14/2010

1. A cell phone company has a base charge of $20 per month. The first 200 minutes are free, and the next 400 minutes cost $0.15 per minute. Usage over 600 minutes costs $0.25 per minute. Find a function, $C(t)$, the amount of a cell phone bill for the month in which a customer uses $t$ cell phone minutes.

$$C(t) = \begin{cases} 
20, & 0 \leq t \leq 200 \\
20 + 0.15(t-200), & 200 < t \leq 600 \\
20 + 0.15(200) + 0.25(t-400), & t > 600 
\end{cases}$$

2. Find the domain of $f(x) = \frac{2x^2}{\ln \sqrt{x^2 - 4}}$.

$x^2 - 4 > 0$
$(x+2)(x-2) > 0$

Domain of $f$: $(0, \infty)$

$X^2$ $X-2$
$+$ $-$ $+$
$-2$ $2$

$(-\infty, -2) \cup (2, \infty)$
3. The per capita consumption of potato chips in a small community increased in the past five years as shown in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>mg per person per year</td>
<td>2.1</td>
<td>2.4</td>
<td>2.6</td>
<td>2.9</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Let \( x = \text{years after 2001} \)

Find a linear model through regression on the calculator.

a) Give the slope and interpret it in the context of the problem.

\[ y = 0.27x + 1.82 \]

b) If the trend continues, how many mg of potato will be expected per capita in 2011 (use the unrounded equation)?

\[ y_1 (?) = 3.99 \]

\[ 3.99 \text{ mg per person in 2011} \]

\[ \text{LinReg}(a x + b) L_1, L_2, Y_1 \]

\[ y = ax + b \]
\[ a = 0.27 \]
\[ b = 2.1 \]
4. Find a function rule for the function f shown below.

\[ f(x) = \begin{cases} 
-x - 3, & x \leq 0 \\
3, & 0 < x \leq 4 \\
2x - 5, & x > 4 
\end{cases} \]

1st piece - slope = -1
y-intercept (0, -3)

2nd piece - horizontal at y = 3

3rd piece - use 2 pts
(4, 3) (6, 7)  
\[ m = \frac{7 - 3}{6 - 4} = 2 \]
\[ y - 3 = 2(x - 4) \]
\[ y - 3 = 2x - 8 \]
\[ y = 2x - 5 \]
5. An object is launched into the air with a velocity of 80 feet per second, with height $h$, after $t$ seconds given by $h(t) = -16t^2 + 80t$ feet.

a) Find the height of the object after 3 seconds.

$$h(3) = -16(3)^2 + 80(3) = -144 + 240 = 96 \text{ ft}$$

b) Find the average velocity between $t = 3$ and $t = 4$.

rate of change $v(4) = h(4) - h(3) = 64 \quad \frac{64 - 96}{4 - 3} = -32 \text{ ft/sec}$

c) Estimate the instantaneous velocity when $t = 3$ by calculating the average velocity over the following time intervals: [3, 3.01], [3, 3.001], and [3, 3.0001].

$$\begin{align*}
3 & \quad h(3) = 96 \\
3.01 & \quad 95.9384 \\
3.001 & \quad 95.9939784 \\
3.0001 & \quad 95.999839884
\end{align*}$$

$$\begin{align*}
\frac{95.9384 - 96}{3.01 - 3} & = -16.16 \\
\frac{95.9939784 - 96}{3.001 - 3} & = -16.016 \\
\frac{95.999839884 - 96}{3.0001 - 3} & = -16.0016
\end{align*}$$

seems to be approaching $-16 \text{ ft/sec}$
6. The half-life of a certain radioactive substance is 350 years.
   
a) Find a function for the amount remaining after $t$ years in a sample of size 80 grams.
   
   \[ A = A_0 e^{-kt} \]
   
   \[ A = 80 \left( \frac{1}{2} \right)^{t/350} \]
   
   \[ A_{120} = 80 \left( \frac{1}{2} \right)^{120/350} \approx 63.0789 \]
   
b) How much is left after 120 years? (Show algebraic solution)
   
   \[ A_{120} = 80 \left( \frac{1}{2} \right)^{120/350} \approx 63.0789 \]
   
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   \[ 80 \left( \frac{1}{2} \right)^{120/350} \approx 63.0789 \]
   
c) When will there be none of the substance left?

7. Evaluate \[ \lim_{x \to 3} \frac{2x^2 - x - 15}{x^2 - 9} \]
   
   \[ = \lim_{x \to 3} \frac{(2x+5)(x-3)}{(x+3)(x-3)} \]
   
   \[ = \lim_{x \to 3} \frac{2x+5}{x+3} \]
   
   \[ = \frac{2(3)+5}{3+3} \]
   
   \[ = \frac{11}{6} \]
8. The graphs of \( f \) and \( g \) are shown below. Use them to evaluate each limit. If the limit does not exist, explain why.

\[
\lim_{x \to 2} [f(x) + g(x)] = \frac{f(2)}{x-3} + \frac{g(2)}{x-3} = \text{DNE} + \text{DNE}
\]

\[
\lim_{x \to 0} f(x)g(x) = \frac{f(0)}{x-3} \cdot \frac{g(0)}{x-3} = 1(-1) = -1
\]

\[
\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{f(0)}{x-3} \cdot \frac{g(0)}{x-3} = \frac{-1}{3}
\]

\[
\lim_{x \to 1} g(x)
\]

Feb 13-11:45 PM
9. If \( f(x) = \sqrt{x - 6} \), \( g(x) = 3e^{2x} \), and \( h(x) = \frac{2}{x-3} \), find \( f \circ g \circ h \) and state its domain.

\[
(f \circ g \circ h)(x) = f(g(h(x))) = f\left(3e^{\frac{2}{x-3}}\right) = \sqrt{3e^{\frac{2}{x-3}}} - 6
\]

\[
3e^{\frac{2}{x-3}} - 6 \geq 0 \implies 3e^{\frac{2}{x-3}} \geq 6 \implies e^{\frac{2}{x-3}} \geq 2 \implies \frac{2}{x-3} = \ln 2
\]

Domain: \((3, \infty)\)

3 \leq x \leq 2 (\approx 1.69314718)
9. If \( f(x) = \sqrt{x - 6} \), \( g(x) = 3e^{2x} \), and \( h(x) = \frac{2}{x-3} \), find \( f \circ g \circ h \) and state its domain.

10. Write the function \( g(y) \) which results from shifting the function \( f(y) = e^y \) to the left _2_ units, then reflecting about the \( x \)-axis, then reflecting about the \( y \)-axis, stretching vertically by a factor of four, and translating up \_9_ units.

\[
g(y) = -4e^{-(y+2)} + 9
\]
11. Find the values of $a$ and $b$ such that the function $f(x)$ is continuous for all real numbers $x$.

\[
\begin{align*}
f(x) &= \begin{cases} 
\frac{e^x}{2x-a} & \text{if } x < 2 \\
\cos x - b & \text{if } x \geq 2
\end{cases}
\end{align*}
\]

\[
f'(x) = \begin{cases} 
\frac{e^x}{2x-a} & \text{if } x < 2 \\
\cos x & \text{if } x \geq 2
\end{cases}
\]

$c^0 = f(0) - a$

\[
2(2) - a = \cos 2 - b
\]

$4 - a = \cos 2 - b$

$4(-1) = \cos 2 - b$

$5 = \cos 2 - b$

$b + 5 = \cos 2$

\[
b = \cos 2 - 5
\]

\[
a = -1
\]

12. Find $\lim_{x \to 7} \frac{\sqrt{x+2} - 3}{\sqrt{x+2} + 3}$.

\[
= \lim_{x \to 7} \frac{x+2 - 9}{(x-7)(\sqrt{x+2} + 3)} = \lim_{x \to 7} \frac{x-7}{(x-7)(\sqrt{x+2} + 3)}
\]

\[
= \lim_{x \to 7} \frac{1}{\sqrt{x+2} + 3}
\]

\[
= \frac{1}{\sqrt{10} + 3}
\]
13. Find \( \lim_{x \to \infty} \frac{x-2}{\sqrt{4x^2 + 1}} \):

\[
\lim_{x \to \infty} \frac{x}{2x} = \lim_{x \to \infty} \frac{1}{2} = \frac{1}{2}
\]

14. Solve for \( y \):

\[
2 \log_2 y - \log_2 (y-3) = \log_{10} 100
\]

\[
\log_2 \frac{y^2}{y-3} = 2
\]

\[
\frac{y^2}{y-3} = 2
\]

\[
y^2 = 2y - 6
\]

\[
y^2 - 2y + 6 = 0
\]

\[
y = \frac{2 \pm \sqrt{4-24}}{2}
\]

\[
y = \frac{2 \pm \sqrt{-20}}{2}
\]

\[
y = \frac{2 \pm \sqrt{-20}}{2}
\text{no solution}