Math 131 Week in Review
Sections 2.6, 2.7, 2.8, 3.1
2/28/10

Finding Derivatives By Definition (Sections 2.6-2.7)

\[ f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \]

1. Find \( f'(a) \) for \( f(t) = t - 2t^3 \)

\[ f'(a) = \lim_{h \to 0} \frac{a+h-2(a+h)^3 - (a-2a^3)}{h} \]
\[ = a+h-2a^3 - (a-2a^3) \]
\[ = \frac{a+h-2a^3}{h} \]
\[ = \frac{a-2a^3}{h} \]

2. Find the equation to the tangent line of \( f(t) \) in \#1 at \( x = 1 \).

\[ f(1) = 1 - 2(1)^3 = 1 - 2 = -1 \]
\[ f'(1) = 1 - 6(1)^2 = 1 - 6 = -5 \]
\[ y+1 = -5(x-1) \]
\[ y = -5x + 4 \]

3. Find \( f'(a) \) for \( f(x) = \frac{3}{x^2} \).

\[ f'(a) = \lim_{h \to 0} \frac{\frac{3}{a+h} - \frac{3}{a}}{h} \]
\[ = \lim_{h \to 0} \frac{\frac{3a - 3a-h}{a(a+h)^2}}{h} \]
\[ = \lim_{h \to 0} \frac{3a - 3a-h}{a^2 h(a+h)^2} \]
\[ = \lim_{h \to 0} \frac{-2a}{a^2 h(a+h)^2} \]
\[ = \frac{-2a}{a^2 (a+0)^2} \]
\[ = \frac{-2a}{a^4} \]
\[ = \frac{-2}{a^3} \]
4. Find $f'(a)$ for $f(x) = \frac{2}{x-3}$.

\[
\lim_{h \to 0} \frac{\frac{2}{a+h-3} - \frac{2}{a-3}}{h} = \lim_{h \to 0} \frac{2(a-3) - 2(a+h-3)}{h(a+h-3)(a-3)}
\]

\[
= \lim_{h \to 0} \frac{2h}{h(a+h-3)(a-3)} = \frac{2}{(a-3)^2}
\]

5. Find the equation to the tangent line for $f(x)$ in #4 at $x = 4$.

\[
f(4) = \frac{2}{4-3} = 2
\]

\[
f'(4) = \frac{-2}{(4-3)^2}
\]

Point: $(4,2)$

Slope: $m = -2$

\[
y - 2 = -2(x - 4)
\]

\[
y - 2 = -2x + 8
\]

\[
y = -2x + 10
\]
6. Find \( f'(a) \) for \( f(x) = \frac{4}{\sqrt{1-x}} \).

\[ \lim_{h \to 0} \frac{4\sqrt{1-a} - 4\sqrt{1-a-h}}{h} = \lim_{h \to 0} \frac{4}{\sqrt{1-a} + \sqrt{1-a-h}} \]

\[ = \lim_{h \to 0} 4 \left( \frac{1-a + (1+h)}{h \sqrt{1-a-h} (\sqrt{1-a} + \sqrt{1-a-h})} \right) = \lim_{h \to 0} \frac{4}{\sqrt{1-a-h} (\sqrt{1-a} + \sqrt{1-a-h})} \]

7. Sketch a graph of a function \( g \) for which \( g(0) = 1 \), \( g'(0) = -2 \), \( g(1) = 2 \), and \( g'(1) = 1 \).
8. A particle moves along a straight line with equation of motion \( s(t) = 50 + 8t - 16t^2 \). Find the velocity and speed when \( t = 3 \).

\[
v(t) = s'(t) = 8 - 32t
\]

\[
v(3) = 8 - 32(3) = 8 - 96 = -88
\]

\[s'(t) = 0 \quad \Rightarrow \quad 8 - 32t = 0 \Rightarrow t = \frac{1}{4}\]

\[s(t) = 0(\frac{1}{4}) + 1.8(\frac{1}{4})^2 = 0 + 1.8 - 0.225 = 1.575
\]

Speed has no direction.

\[|v(t)| = |8 - 32t| = |8 - 96| = |-88| = 88\]
Derivatives From a Table (Section 2.6)

9. The table below gives the approximate distance traveled by a downhill skier after $t$ seconds for $0 \leq t \leq 10$. What is the meaning of $D'(6)$? Estimate its value.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>0</td>
<td>13</td>
<td>53</td>
<td>120</td>
<td>212</td>
<td>333</td>
</tr>
</tbody>
</table>

meters distance

rate of change of distance over time (velocity) at 6 seconds

\[
\frac{120-63}{6-4} = \frac{57}{2} \text{ m/sec} \quad \frac{212-120}{8-6} = \frac{92}{2} = 46 \text{ m/sec}
\]

\[
\frac{1}{2} (\frac{57}{2} + 46) = 39.75 \text{ m/sec}
\]

10. The graphs of $g$, $g'$, and $g''$ are given below. Label the graphs appropriately.

$g'$ gives the slope of $g$

green is slope (derivative) of red
red is slope (derivative) of blue
Derivatives From a Graph (Section 2.7)

11. Estimate the slope of the tangent at each of the points (dots) on the graph.

12. Sketch the derivative of $f$ on the same coordinate system above.
13. Estimate the slope of the tangent at each integer x-value on the graph.

14. Sketch the derivative of f on the same coordinate system above.
15. Give the intervals or points where the graph given below is not differentiable. Explain.


Cusp (pointed)
Discontinuous
Vert: ext. tangent lin

\( x = -1 \) Cusp
\( x = 0 \) Discontinuous
\( x = 2 \) Discontinuous
endpoint \( x = -2, x = 3 \)
16. Sketch a graph of a function that satisfies all of the given conditions.

\[ F'(1) = F'(3) = 0 \quad \text{horizontal tangent} \]

\[ F'(x) > 0 \text{ if } x < 1 \text{ or } x > 3 \quad \text{positive slope} \]

\[ F'(x) < 0 \text{ if } 1 < x < 3 \quad \text{negative slope} \]
17. Differentiate: \( g(x) = x \sqrt{29} = \sqrt{29} \cdot x \)
\[
g'(x) = 1 \sqrt{29} \cdot x^0 = \sqrt{29}
\]

18. Differentiate: \( f(t) = t^3 + 5t^2 - \frac{2}{3} t + 21 \)
\[
f'(t) = 3t^2 + 2(5)t' + 1\left(-\frac{2}{3}\right)t^0 + 0
= 3t^2 + 10t - \frac{2}{3}
\]

19. Find any and all \( x \)-values where the tangent line is horizontal.
\( \text{Slope} = 0 \quad f'(t) = 0 \)
\[
9t^2 + 10t - \frac{2}{3} = 0
\]
\[
9t^2 + 90t - 2 = 0
\]
\[
t = \frac{-30 \pm \sqrt{900 + 72}}{18}
= -30 \pm \sqrt{972}
= -30 \pm 18\sqrt{3}
= \frac{-30 \pm 18\sqrt{3}}{18}
= \frac{-5 \pm 3\sqrt{3}}{3}
\]
20. Differentiate: \( h(x) = \frac{3x^2 - 6x + 24}{3x} \)

\[
\begin{align*}
    &= \frac{3x^2}{3x} - \frac{6x}{3x} + \frac{24}{3x} \\
    &= x - 2 + 8x^{-1}
\end{align*}
\]

\( h'(x) = 1 + (-1)8x^{-2} \)

\[
\begin{align*}
    &= 1 - 8x^{-2} \\
    &= 1 - \frac{8}{x^2} \quad \text{or} \quad \frac{x^2 - 8}{x^2}
\end{align*}
\]
21. Differentiate: \( f(u) = e^{u-1} + 3 \)
\[
\frac{d}{dx} e^x = e^x
\]
\[
\frac{d}{dx} (e^{u-1} + 3) = e^{u-1} \quad f'(u) = e^{u-1}
\]

22. Find the equation of the tangent line for the function in #21, at \( u = 1 \).
\[
f'(1) = e^{1-1} = e^0 = 1
\]
\[
f(1) = e^{1-1} + 3 = 0 + 3 = 1 + 3 = 4
\]
\[
\text{point } (1,y)
\]
\[
slope = 1
\]
\[
y - y = 1(x - 1)
\]
\[
y - (4) = x - 1
\]
\[
y = x + 3
\]

23. Differentiate: \( g(t) = \sqrt[3]{t} - 5\sqrt[3]{t^3} \)
\[
g(t) = t^{\frac{1}{3}} - 5t
\]
\[
g'(t) = \frac{1}{3} t^{-\frac{2}{3}} - \frac{3}{2} (5)t^{\frac{2}{3}}
\]
\[
= \frac{1}{3} t^{-\frac{2}{3}} - \frac{15}{2} t^{\frac{2}{3}}
\]
\[
= \frac{1}{3} \frac{\sqrt[3]{t}}{t^2} - \frac{15\sqrt[3]{t}}{2}
\]
\[
= \frac{\sqrt[3]{t}}{3t} - \frac{15\sqrt[3]{t}}{2}
\]
\[
= \frac{\sqrt[3]{t}}{3t} - \frac{15\sqrt[3]{t}}{2}
\]
\[
\frac{\sqrt[3]{t}}{3t} - \frac{15\sqrt[3]{t}}{2}
\]
\[
\frac{\sqrt[3]{(3t)(t)}}{3t} - \frac{15\sqrt[3]{(3t)(t)}}{2(3t)}
\]
\[
\frac{2\sqrt[3]{t} - 45\sqrt[3]{t}}{6t}
\]