Math 131 Week in Review
Sections 2.6 - 2.8, 3.1 - 3.4, 3.7 – 3.9
3/21/10

1. Use the definition to find the derivative of \( f(x) = \frac{2x - 3}{x + 1} \).

2. Find the equation of the tangent line and the normal line to the following curve when \( t = 0 \).
   \[ y = (3t^2 - 2t + 1)(t - \cos t) \]

3. A piece function is defined as follows:
   \[ F(x) = \begin{cases} 
   x + 1 & x < 0 \\
   e^x & 0 \leq x < 1 \\
   2x^2 - 1 & x \geq 1 
   \end{cases} \]

   a) Find any intervals where \( F \) is discontinuous.

   b) Find any intervals where \( F \) is not differentiable.
4. Find the derivative of the following functions:

a) \( G(x) = \tan(e^x - 2x) \)

b) \( H(x) = \frac{e^{2x} + e}{e^{2x} - e^{-x} + 2} \)

c) \( F(x) = \ln(2x^3 - 5)^2 \)

5. The temperature (in degrees Fahrenheit) of a person during an illness can be modeled by \( T = -0.0375t^2 + 0.3t + 100.4 \), where \( t \) is time in hours since the person started to show signs of a fever.


a) Find \( dT/dt \) and explain its meaning in this situation.

b) Find the rate of change of body temperature after 4 hours.

c) Over what interval(s) is the function increasing?

d) Over what interval(s) is the function concave down?
6. Where does the function given below have a horizontal tangent line or lines over the interval \([0, 2\pi]\)?

\[ F(x) = \cos x - \sin x \]

7. The function \( g(t) = \frac{t^2 - t + 1}{t^2 + 1} \) measures the level of oxygen in a pond, where \( t \) is the time (in weeks) after organic waste is dumped into the pond.


Find the rate of change of \( g \) with respect to \( t \) when \( t = .5 \).

8. The predicted cost \( C \) (in hundreds of thousands of dollars) for a company to remove \( p\% \) of a chemical from its waste water can be modeled by

\[ C = \frac{124p}{(10 + p)(100 - p)}, \quad 0 \leq p < 100. \]


a) Find the rate of change in the cost when \( p = 25\% \).

b) Find the rate of change in the cost when \( p = 75\% \).

c) What happens to \( C \) and \( C' \) as \( p \) approaches 100?
9. An environmental study indicates that the average daily level $P$ of a certain pollutant in the air (in parts per million) can be modeled by the equation
\[ P = 0.25\sqrt{0.5n^2 + 5n + 25} \]
where $n$ is the number of residents of the community (in thousands). Find the rate at which the level of pollutant is increasing when the population of the community is 12,000.

10. The number of children enrolled $E$ (in thousands per year) in the State Children’s Health Insurance Program (SCHIP) for 1998 through 2005 can be modeled by
\[ E = \frac{21892 - 2812.9t}{1 - 0.2759t}, \]
where $t$ is the time in years, with $t = 8$ corresponding to 1998.

   a) Find the average rate of change from 2000 through 2005.

   b) Find the instantaneous rate of change for 2005.

11. The normal daily maximum temperature $T$ (in degrees Fahrenheit) for a Midwest city can be modeled by
\[ T = \sqrt{5.9918t^4 - 171.015t^3 + 1469.25t^2 - 3560.6t + 4292}, \]
where $t$ is the time in months, with $t = 1$ corresponding to January.

Find the rate of change in March.
12. The following is a graph of a function $g$, along with its first and second derivative. Identify which is $g$, $g'$, and $g''$.

13. Estimate the derivative of $f$ at $x = 2$, given the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>5.5</td>
<td>2.3</td>
<td>4.1</td>
<td>3.5</td>
<td>5.1</td>
</tr>
</tbody>
</table>

14. Given the graph of $h$ below, sketch $h'$ on the same coordinate system.
15. Given the position function \( s(t) = x^3 - 2.5x^2 - 2x - 4.2 \), find the acceleration when the velocity is zero.

16. Given the graph of \( G' \) below, give the intervals where \( G \) is increasing, decreasing, concave up, and concave down.

17. Given \( f(x) = \frac{1}{1-x} \), find the linear approximation near the point \( a = 0 \).
Formulas to Know

\[ \frac{d}{dx} (c) = 0 \quad \frac{d}{dx} (x) = 1 \quad \frac{d}{dx} (x^n) = nx^{n-1} \quad \frac{d}{dx} (e^x) = e^x \quad f'(x) = f'(0)a^x \]

\[ (f \pm g)' = f' \pm g' \quad (cf)' = cf' \quad (fg)' = fg' + gf' \quad \left( \frac{f}{g} \right)' = \frac{gf' - fg'}{g^2} \]

\[ \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \quad \lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = 0 \quad \frac{d}{dx} (\sin x) = \cos x \]

\[ \frac{d}{dx} (\cos x) = -\sin x \quad \frac{d}{dx} (\csc x) = -\csc x \cot x \]

\[ \frac{d}{dx} (\sec x) = \sec x \tan x \quad \frac{d}{dx} (\tan x) = \sec^2 x \quad \frac{d}{dx} (\cot x) = -\csc^2 x \]

\[ (f \circ g)'(x) = f'(g(x)) \cdot g'(x) \quad \frac{d}{dx} (a^x) = a^x \ln a \quad f''(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

\[ \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad v(t) = s'(t) = \frac{ds}{dt} \quad a(t) = v''(t) = s''(t) \]

\[ \frac{d}{dx} (\ln x) = \frac{1}{x} \quad \frac{d}{dx} |x| = \frac{1}{x} \]

\[ y = f(a) + f'(a)(x - a) \quad dy = f'(x)dx \quad f(a + dx) \approx f(a) + dy \]