Math 131 Week in Review
Sections 4.2, 4.3, 4.6
4/4/10

1. Sketch a graph of a function \( f \) that is continuous on \([-2, 3]\), has an absolute minimum at -1, an absolute maximum at 3, and a local minimum at 0.

A critical number of a function \( f \) is a number \( c \) in the domain of \( f \) such that either \( f'(c) = 0 \) or \( f'(c) \) does not exist.

2. Sketch the graph of a function that has 2 local minima, 1 absolute maximum, and 6 critical numbers.
3. Find the critical numbers of \( g(x) = 2 \cos x - \sin^2 x, \ 0 \leq x < \pi. \)

4. Find the critical numbers of \( h(x) = x^3 - 3x^2 - 4x. \)

5. Find the critical numbers of \( F(x) = x^2 e^{-4x}. \)

6. Find the critical numbers of \( G(x) = \frac{2}{x^3} (x - 3)^2. \)

7. Find the absolute maximum and absolute minimum value of \( f(x) = x^4 - 3x^2 + 2 \) on the interval \([-2, 3]\).
8. Find the absolute maximum and absolute minimum value of \( G(x) = \frac{x^2 - 9}{x^2 + 9} \) on the interval \([-5, 5]\).

9. What is the smallest perimeter possible for a rectangle whose area is 16 in\(^2\), and what are its dimensions?
   

10. Two sides of a triangle have lengths \( a \) and \( b \), and the angle between them is \( \theta \).
    
    What value of \( \theta \) maximize the triangle’s area? [Hint: \( A = \frac{1}{2} ab \sin \theta \).]
    
11. The height of an object moving vertically is given by \( s = -16t^2 + 96t + 112 \), with \( s \) in ft and \( t \) in sec.

Find
i. the object’s velocity when \( t = 0 \),

ii. its maximum height and when it occurs, and

iii. its velocity when \( s = 0 \).

12. Jane is 2 mi offshore in a boat and wishes to reach a coastal village 6 mi down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time?

14. A rectangle is to be inscribed on the x-axis under the arch of the curve \( y = -x^2 + 3 \). What are the dimensions of the rectangle with largest area, and what is the largest area?