The Secant Conjecture in the Real Schubert Calculus

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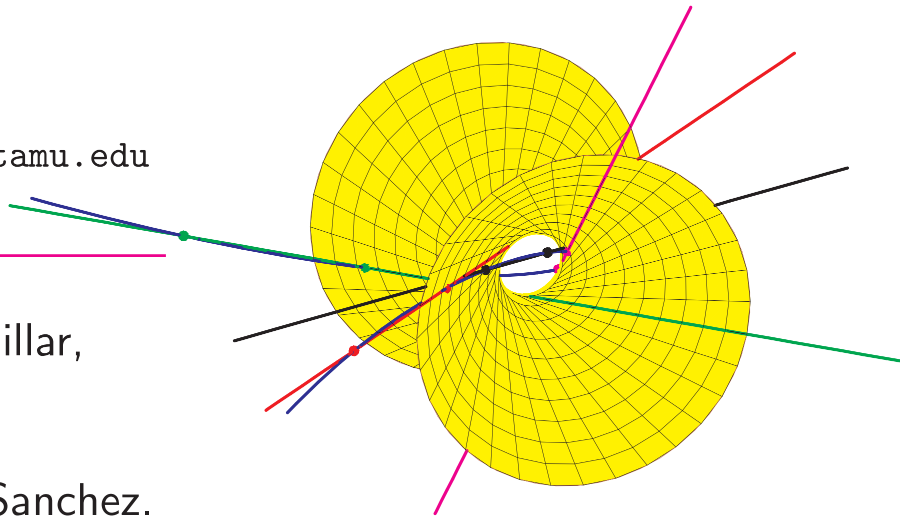

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Shapiro Conjecture in Schubert calculus

Schubert calculus is an important class of geometric problems involving linear spaces.

A *Schubert variety* $X_v F_\bullet$ is a set of linear spaces satisfying a condition v imposed by a flag $F_\bullet$.

Eg. the set of lines in space that meet a point.

Schubert problem: Conditions $v_1, \ldots, v_s$ s.t. $\bigcap_{i=1}^{s} X_{v_i} F_\bullet^i$ is finite.

Shapiro Conjecture. For any Schubert problem $v_1, \ldots, v_s$, if $F_\bullet^1, \ldots, F_\bullet^s$ are real flags osculating a rational normal curve, then $\bigcap_{i=1}^{s} X_{v_i} F_\bullet^i$ is transverse and all points are real.

Shapiro Conjecture: status

The Shapiro conjecture makes sense for all flag manifolds

Grassmannians: True for $Gr(n-2, n)$ (Eremenko-Gabrielov, Annals 2002), and in general (Mukhin-Tarasov-Varchenko, Annals, to appear).

Type A flag manifolds: False, but in an interesting way. Entensive computation suggested the Monotone Conjecture, an appealing correction.

Lagrangian Grassmannian: Also interestingly false, but with possible correction involving Levi-movability.

Orthogonal Grassmannian: No counterexamples, yet.

Other flag manifolds: ????

Secant Conjecture

Eremenko, et. al. proved a result about rational functions that implies the Monotone Conjecture for flags $E_{n-2} \subset E_{n-1} \subset \mathbb{C}^n$.

In terms of Schubert calculus on $Gr(n-2, n)$, this suggests a new generalization of the Shapiro Conjecture.

A flag $F_\bullet$ is secant to the rational normal curve γ along an interval $I \subset \gamma$ if each subspace in $F_\bullet$ is spanned by its intersections with I.

Secant Conjecture. For any Schubert problem $v_1, \ldots, v_s$ on a Grassmannian, if $F_\bullet^1, \ldots, F_\bullet^s$ are secant to a rational normal curve along disjoint intervals, then $\displaystyle\bigcap_{i=1}^{s} X_{v_i} F_\bullet^i$ is transverse and all points are real.

Secant Conjecture implies the Shapiro Conjecture for Grassmannians.

Secant Conjecture in pictures

View Animation

Experimentation for Secant Conjecture

We are studying the Secant Conjecuture computationally.

Goal: Test as many instances of the conjecture as possible, in particular, all Schubert problems on all small Grassmannians.

Chris Hillar designed a general framework for this, organizing the computation via databases, monitoring it from web pages.

We are using a super computer: a 1.1 teraflop Beowulf cluster whose day job is Calculus instruction.

Currently, used >110 GHz-years of computing, studying >490 million geometric Schubert problems.

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