Math 151H
Sections 201 and 202
Third Test

28 Novemer 2006
Instructor: F. Sottile

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Full credit is given only for complete and correct answers.
No aids allowed on the exam. Please write your answers in blue books.
Do persevere; partial credit will be given, and you are all good students.
Point totals are in brackets next to each problem.

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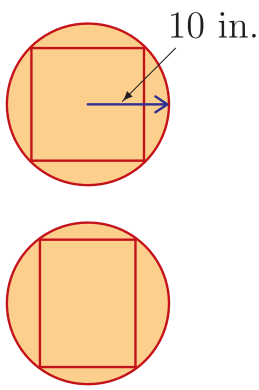
1. [10] State one version of the Fundamental Theorem of the Calculus.

2. [5] Using the Fundamental Theorem of the Calculus, give a formula for a function $F(\eta)$ whose derivative with respect to $\eta$ is $\sqrt{1+\sin\sqrt{\eta}}$.

3. [10] Find the area between the $x$-axis and the curve $y = 16 - x^4$.

4. [20] A rectangular beam is to be cut from a cylindrical log of radius 10 inches.
   (a) Show that the beam of maximal cross-sectional area is a square. (Do not formulate this with trigonometric functions.)
   (b) Find the dimensions of the strongest beam that can be cut from this log, if the strength of a rectangular beam is proportional to the product of its width and the square of its depth.




5. [20] Calculate the following limits,

$$\lim_{\xi\to 0} \frac{\sin^3(\xi)}{\sin(\xi^3)} \qquad \text{and} \qquad \lim_{\zeta\to 0^+} \frac{\sqrt{1-\cos\zeta}}{\sin\zeta} .$$

6. [15] Find the extreme values of the function $f(\tau) := \tau - \sqrt{2}\sin\tau$ for $\tau$ in the interval $[\pi, \pi]$. Sketch the graph of the function and give the intervals over which it is increasing and over which it is decreasing.

7. [10] While bicycling into school one day, Frank passes the same SUV twice. Prove that at sometime on the way in the two vehicles had the same acceleration.

8. [10] Let $f$ be a function defined on the interval $[\alpha, \beta]$. Indicate whether each of the following statements is true or false.
   (a) If $f$ is differentiable and increasing on $(\alpha, \beta)$, then $f'(\gamma) > 0$ for all $\gamma \in (\alpha, \beta)$.
   (b) If $f$ is increasing on $(\alpha, \beta)$, then the function $F(\kappa) := \int_\alpha^\kappa f(t)dt$ is increasing.
   (c) If the function $f$ acheives its maximum value on the interval $[\alpha, \beta]$ at a point $\mu \in (\alpha, \beta)$, then $f'(\mu) = 0$.

Ω. [5 pts extra credit] State the *other* version of the Fundamental Theorem of the Calculus.