Math 151H
Sections 201 and 202
Final exam

10 December 2008
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Full credit is given only for complete and correct answers.
No aids allowed on the exam. Please write your answers in blue books.
Do persevere; partial credit will be given, and you are all good students.
Point totals are in brackets next to each problem.

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1. (a) [10] Suppose that f is a function and l, a are real numbers. Give the precise ϵ–δ definition of *limit*. That is, give the definition of: "The function f approaches the limit l near a".

   (b) [20] Using this definition of limit, prove that $\lim_{x \to 4} \left(\frac{1}{x} \right) = \frac{1}{4}$.

2. [15] Suppose that $f(x) = x^3 + x - 1$. Use a Theorem from the class to show that $f(x) = 0$ has a solution in the interval $[0, 1]$. Does it have any more solutions? (Why or why not?)

3. [40] Compute the derivatives with respect to the variable x of the following functions.

   (a) $\log_5(x)$

   (b) $\sin(e^{x^2})$

   (c) $\sin(\arctan(x))$

   (d) $x^2 + 5x^{\sqrt{2}}$

   (e) $x^{\sinh(x)}$

   (f) $\sin(x^2 + \sin(x^2 + \sin(x^2)))$

   (h) $\sin\left(\frac{\cos x}{x} \right)$

   (i) $\frac{\sin(x^2) \sin^2 x}{1 + \sin x}$

4. [15] Prove the identity.

$$\tanh(\ln x) = \frac{x^2 - 1}{x^2 + 1}.$$

5. [20] Let $f(x) := x^4 - 2x^2 - 1$. Find the relative extrema and inflection points of f and determine the intervals on which it is increasing, decreasing, and has constant concavity.

6. [20] A particle is moving in the xy-plane in such a way that its position at time t is $\mathbf{r}(t) = (1 + 3\cos t)\mathbf{i} + 2\sin t\mathbf{j}$. Find the particle's maximum speed.

7. [25] A street light is at the top of a 5m tall pole. A man 2m tall walks away from the pole with a speed of 2m/sec along a straight path. How fast is the tip of his shadow moving when he is 13m from the pole? How fast is his shadow lengthening at that point?

8. [10] Find y' when $x^4 + y^5 - 6x^2y^3 = 5$.

9. [10] Using only methods from this course, compute the following limit

$$\lim_{x \to a} \frac{\sin x - \sin a}{x - a} .$$

10. [15] State both versions of the Fundamental Theorem of the Calculus, defining your terms (e.g. what is f?) and using complete sentences (with formulas, of course).