A General View of Mathematics before 1000 BCE[1]

1 Where did Mathematics Start?

We have considered some very early examples of counting. At least one dated to 30,000 B.C. Counting is but the earliest form of mathematics. It was first a simple device for accounting for quantity. However, this is so basic, even primitive, that it cannot be considered as a subject or science.

We are looking for sources of mathematical thought in human activity. These sources come in the form of carvings, inscriptions or manuscripts. Evidence of this kind have four countries of origin, all dating to similar times. They are:

- India
- Egypt
- Mesopotamia
- China

What are the commons features of all?

- Each claims a high degree of antiquity.
- Each claims to have been a pioneer in mathematical development.
- Each had a relatively warm climate, with fertile lands.
- Each flourished along major rivers.
- Each had a strong centralized government and a strong religious life.

In this chapter, we take up all too briefly the contributions of very early China and India, with separate chapters devoted to the mathematics of Egypt and Babylon. To be sure, the records for the very early mathematical contributions are not known with the same accuracy as those for Greece, for example. Thus much of what follows is nearer to speculation than to established historical fact.

2 China

Some authors have argued that systematic astronomical observations, such as identifying and naming the constellations, occurred in China as early as 17000 BCE, which of course is just at the close of the Early Stone Age. Moreover, 15,000 BCE is the earliest date given to the creation of the zodiac. If so, even if the date is accurate to within a few millenia, this places astronomy as the first scientific subject and China well in advance of any other peoples. Other historians give much later dates such as 4000 BCE for these beginnings, making the origin more or less contemporary with Egypt and Babylon. The reason for tagging astronomy to mathematics is the clear connection of accurate mensuration of angles and use of large numbers involved in the study.

More credible evidence places writings on astronomy during the reign of Huang-ti[2] which began in 2697 BCE. Bearing in mind the uncertainty of historical correctness, it is reported that during his reign Li Shu wrote on astronomy and Ta-nao established the sexagesimal system.

The earliest of the Chinese classics is the *I-king* (or *Book of Permutations*. In it we find symbols (– and --), the *yang* and the *yin*, respectively, which seem to have been used in permutations in a primitive binary system. Similar symbols are used in the *Pa-kua*, where interpreting the – and -- as the

[2]Shen Yen Huang-ti, the "Yellow emperor", was third of ancient China's mythological emperors, a vigorous soldier- emperor, culture hero and a patron saint of Taoism. Huang-ti is reputed to have been born about 2704 BCE. Among his many accomplishments, he brought to China the bow and arrow, wooden houses, and writing. Others give these accomplishment to the previous emperor, Fu Hsi (c. 2852 BCE) and to Huang-ti the introduction of brick houses, official historians, and an astronomical observatory.

numbers 1 and 0 respectively, we see the permutations of these symbols taken three at a time are found. Interpreting these as binary numbers, the values 0 through 8 can be identified. Other identifiable mathematics in the *Pa-kua* are magic squares, also types of permutations.

4	9	2
3	5	7
8	1	6

from the *Pa-kua*

Magic squares seem to be timeless, occuring frequently throughout mathematics history. Historically, they are square matrices divided into cells, filled with letters or numbers letters in special arrangements. They were once thought to have magical properties. Originally they were used as religious symbols but later became charms or tools for divination; Finally, they are now in the realm of curiosities and puzzles, except for some mathematicians who continue to study them in the context of number theory.

It seems believable that a matrix arrangement of numbers to add to the same sum on every column or row might be considered of potence as long as the methodology for its construction is secret and not understood. In this way a mathematical construct enters culture on the mystical side, not the scientific or analytical side we are accustomed to believe. Pythagorean triples may also once have been considered in this light. Indeed they are still remarkable, so much so that one of the great mathematical quests of our epoch was the attempt to discover such "triples" with respect to other powers. Of course, this was Fermat's theorem — to be considered in detail later — and has been resolved in the negative. For our purposes, an arithmetic *magic square* of order n is a square $n \times n$ array filled with the numbers $1, \ldots, n^2$, with each value appearing just once and so that the sums of each row equals the sum of each column. This value is $\frac{n^3+n}{2}$.

Interest in magic squares spread from China to Japan, India, and

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

the Middle East, and they were introduced to Europe in Byzantine times. The first magic square of 4 in the first century in India by a mathematician named Nagarajuna. The first magic squares of 5 and 6 appear in an encyclopedia in Baghdad about 983 AD by Ikhw'n al-Saf' Ras'il, though other earlier Arab mathematicians also wrote about magic squares. The magic square above appears also in an engraving entitled *Melancholia* made in about 1514 by Albrecht Dürer (1471-1528). (See, http://www-groups.dcs.st-and.ac.uk/ history/ Miscellaneous/Durer/Melancholia.html for a picture of this engraving.)

There are many ways to construct magic squares. For example there are eight ways to construct a 3×3 magic square containing the numbers 1 to 9. Shown to the right is a 4×4 magic square. Precisely, a magic square is a square array, say $n \times n$, with the numbers $1, 2, \ldots, n^2$ arranged in the cells so that all the rows and columns have a fixed sum, a *magic* sum. There is no known method to construct all magic squares of a given size, but there are many very interesting algorithms to construct a given one. Other magic squares, some without consecutive numbers, were known to the ancient Chinese as well.

Here is a method for constructing magic squares of prime[3] size. We also use the mod notation to express the remaider of a divided by n equals b. In the mod notation we write this as $b = \text{mod}(a, n)$. For examples, we have $2 = \text{mod}(7, 5)$, $6 = \text{mod}(27, 7)$, and $2 = \text{mod}(-3, 5)$.

Theorem 1 *Suppose that n is a prime and p and q are integers $1 \le p \ne q < n$. For $0 \le i,\ j < n$ define*

$$a_{i+1,\ j+1} = n[(mod(i - pj, n)] + mod(i - qj, n)$$

[3]Recall that a *prime* number is any integer that is divisible only by 1 and itself. A *composite* number is any number divisible by an integer other than 1 and itself. For example, the first few primes are 2, 3, 5, 7, 11, and 13. The first few composite numbers include 4, 6, 8, 9, 10, 12, and so on.

Then the array $A = a_{ij},\ 1 \le i, j \le n$ *is a magic square of order* n.

For example, with $n = 7,\ p = 1$, and $q = 6$, the magic square below results.

$$A = \begin{bmatrix} 3 & 24 & 20 & 11 & 7 \\ 9 & 5 & 21 & 17 & 13 \\ 15 & 6 & 2 & 23 & 19 \\ 16 & 12 & 8 & 4 & 25 \\ 22 & 18 & 14 & 10 & 1 \end{bmatrix}$$

Here is a line of Maple code to generate magic squares. Select $n,\ p$ and q.

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n:=5; A:=array(1..n,1..n);p:=1;q:=n-1;for i from 1 to n do
for j from 1 to n do A[i,j]:=n*(modp(i-p*j,n))+modp(i-q*j,n)+1;od;od;
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As should be evident we require that $p \neq q$ to guarantee that no value is repeated in the array. However with $p = q$, the sum condition still holds. Try to discover what additional conditions should be imposed on p and q in the case that n is composite.

In 213 BCE Shïh Huang-ti (259 - 210(1) BCE), a military emperor who with blood and iron forged the union of the Chinese states under one rule and mounted the throne of all China. Among his achievements was the construction of the Great Wall, a structure 1500 miles long with massive gateways in the Assyrian style. Said Voltaire, beside it, "the pyramids of Egypt are only puerile and useless masses." In an attempt to rewrite history first by destroying it, Shïh Huang-ti, on the advise of his Prime Minister, Li Ssü, ordered all history books burned. Scholars who did not comply with this order were sent to work on the Great Wall, though four hundred and sixty were executed. Scientific books seem to have escaped this sentence of fire, though many were probably lost due to official exuberance. It is unlikely that such an extensive order could be fully carried out. Nonetheless, a vast treasure of Chinese learning was lost forever. The contents of the best of these books were certainly carried in the memories of many surviving scholars for later transcription. The brutality of his life was sustained even in his death, with many souls buried alive with him in a lavish tomb, some being maidens to keep him company in death and others being workmen to keep silent their knowledge of the secret passage to his burial chamber. Almost immediately after his death, the burned books freely circulated once again. Needless to say, Chinese historians were for some time afterwards unsympathetic to his reign. The people were less comforting, defacing his grave for generations.

We can be reasonably certain of the correctness and accuracy of the first authentic mathematics text, the *Chóu-peï*, which dates from about 1105 BCE. Its author is unknown. The *Chóu-peï* contains its mathematics in the form of several dialogues. One for example relates number mysticism, mensuration, and astronomy. Here are a few extracts:

- The art of numbers is derived from the circle and the square.
- Break the line and make the breadth 3, the length 4; then the distance between the corners is 5.
- Forms are round or pointed; numbers are odd or even. The heaven moves in a circle whose subordinate numbers are odd; the earth rests on a square whose subordinate numbers are even.

It also contains an early specimen of the Pythagorean Theorem, but gives no proof.

The greatest of the Chinese classics in mathematics, and the next in order of antiquity, is the *K'iu-ch'ang Suan-shu*, or *Arithmetic in Nine Sections*. Its author and date are unknown. However, after the great book burning, there appeared a mathematician by the name of Ch'ang Ts'ang. He collected great works and appears to have edited the *K'iu-ch'ang Suan-shu*. This book contains nine sections.

1. *Squaring the farm.* Surveying, with correct formulas for areas of triangles, trapezoids, and circle ($\frac{1}{2}c \cdot \frac{1}{2}d$ and $\frac{1}{4}c \cdot d$)[4]
2. *Calculating the cereals.* Percentages and proportions.
3. *Calculating the shares.* Relating to partnership and the Rule of Three.

[4]Curiously, the first solid proof of this result was given by Archimedes, long after *The Elements* appears. The proof, which we will see alter, involves the ancient method of taking limits, called then the method of *exhaustion*.

4. *Finding length.* Finding the sides of figures and including square and cube roots.
5. *Finding volumes.*
6. *Alligation.* Relating to motion problems.
7. *Excess and deficiency.* Relating to the Rule of False Position.
8. *Equation.* Solving simultaneous linear equations, with some notion of determinants.
9. *Right triangle.* The Pythagorean triangle.

3 India

Early Hindu mathematics was produced by a very much different type of people. India's work in science is at once old and new. Young as a secular pursuit, and old as the core of Hindu life was religion. It was to religion that mathematics was the servant. (Such was also the case in the West, as during the Medieval period the art of computation was taught particularly for the calculation the date of Easter[5].) Astronomy grew from the worship of the stars, and their observation was put to the service of feast day, planting and other necessities of life. In this respect mathematics in India was similar to that of Egypt. And as in the Middle Ages of the western world, the scientists of India were her priests. Whatever the origins, and whatever the purpose, the Hindus were generally highly imaginative, and their mathematics developed along such lines as the theory of numbers, geometry, and astronomy. They developed complex computational methods that was mathematically the rival of Greece in every aspect except geometry. The Hindu mind was primarily occupied with the *arithmetical.* Among the most profound parts of the Hindu heritage were the "Arabic numerals"[6], and the decimal system. Though precise origins are obscure there is evidence that it came from India through the Arabs.

[5] Paul Abelson, The Seven Liveral Arts, A Study in Mediaeval Education, New York, 1906: AMS, 1972.

[6] The "Arabic" numerals are found as early as 256 BCE, more than a millennium before their introduction in Arabic literature.

The dominant Hindu mathematicians were Aryabhata (476 - 550), Brahmagupta (c. 598 - c. 670), and Bhaskara (1114 - c. 1185), but their respective dates are far more recent than this chapter allows. Therefore, in the following we give a brief (and inadequate) review of early Hindu mathematics. No doubt this mathematics is very old. However, in the form it comes to us there appears to be a definite Greek influence.

The history of Hindu mathematics may be resolved into two periods: First, the *Súlvasutra period* which terminates not later than 200 A.D., and the *astronomical and mathematical period*, extending from 400 A.D. to 1200 A.D. The term *Súlvasutra* means the "rules of the cord", and originally explained the construction of sacrificial alters. The Súlvasutras were composed sometime after 800 B.C. Their aim was primarily not mathematical but religious. Mathematical parts refer to geometrical ideas and mensuration.

The dating and origin of early Hindu mathematical works is even less certain than the Chinese. Some claims are preposterous. For example, the first edition of the *Surya Siddhanta*, or *Knowledge from the Sun*, of the Swami Press at Meerut, claims the work was compiled 2,165,000 years ago. Other works are dated even earlier. In fact this famous work was probably composed in the 4th or 5th century of our own era.

About all we can say is that there is some evidence from ancient literature that in very early times India was cognizant of calculations, of astronomy, and of geometry. Judging by the nature of their architecture, there must have been some considerable body of "applied arithmetic".

For more information, see *The Crest of the Peacock: The Non-European Roots of Mathematics* by George Gheverghese Joseph, Princeton University Press