Electromagnetic field quantization in the presence of a medium

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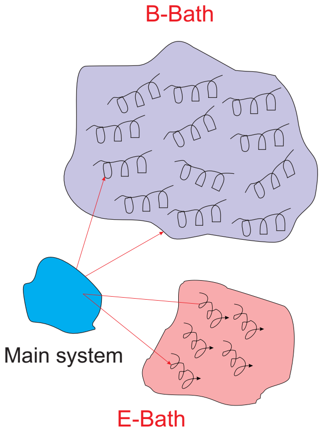
Why we quantize EF in the presence of a medium?

For example in the following problems

- Spontaneous emission of atoms close to dielectric surfaces,
- Energy level shifts of atoms close to dielectrics,
- Static and dynamical Casimir effects,
- Propagation of light pulses through a magneto-dielectric medium,
- Optical properties of nano-structures, etc.

Main idea: Modeling the medium with harmonic oscillators

Hopfield [1], Caldeira-Legget [2, 3], Huttner-Barnett [4], K[5, 6].



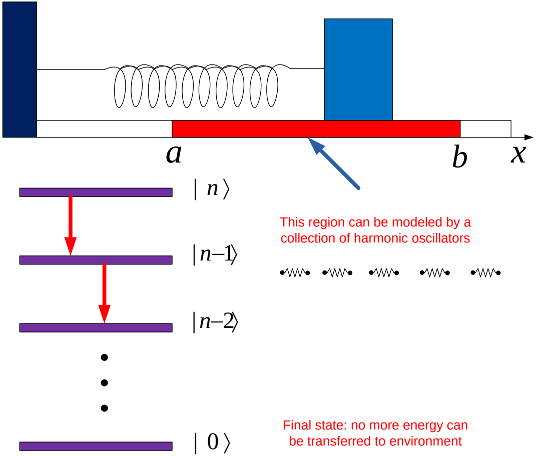
Generality of the approach

- It can be applied to a general field theory (Scalar, Vector, Tensor, Spinor) in the presence of a medium or external potentials K[7, 8].
- It can be applied to a general system in the presence of dissipative or amplifying media K[9].
- It can be applied to nonlinear media K[10]
- Radiation process like Cherenkov radiation K[11]

Methods of open quantum system theory:

- Quantum Langevin equation [12]
- Lindblad super operator method [13]
- Master equation method [14]
- Path-integral method [15]

The quantum damped harmonic oscillator K[16]



Harmonic oscillator

$$L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega_\circ^2 x^2 + \frac{1}{2}\int_0^\infty d\omega\,[\dot{Y}_\omega^2 - \omega^2 Y_\omega^2] + \underbrace{\int_0^\infty d\omega\, f(x,\omega) Y_\omega}_{Polarization}\,\dot{x}$$

$$p = \frac{\partial L}{\partial \dot{x}} = \dot{x} + \int_0^\infty d\omega\, f(x,\omega) Y_\omega$$

$$P_\omega = \frac{\partial L}{\partial \dot{Y}_\omega} = \dot{Y}_\omega$$

$$[x, p] = i\hbar, \quad [Y_\omega, P_{\omega'}] = i\hbar\delta(\omega - \omega')$$

Harmonic oscillator

$$\ddot{\hat{x}} + \omega_\circ^2 \hat{x} + \partial_t \underbrace{\int_0^\infty d\omega f(x,\omega) Y_\omega}_{P} = 0$$

$$\ddot{\hat{Y}}_\omega + \omega^2 \hat{Y}_\omega = f(x,\omega)\dot{\hat{x}}$$

$$\hat{Y}_\omega = \underbrace{\sqrt{\frac{\hbar}{2\omega}}(\hat{a}_\omega e^{-i\omega t} + \hat{a}_\omega^\dagger e^{i\omega t})}_{\text{Noise or fluctuating field}} + \int_{-\infty}^{t} dt' \underbrace{\frac{\sin\omega(t-t')}{\omega}}_{\text{Green's function}} f(x,\omega)\dot{\hat{x}}(t')$$

$$[\hat{a}_\omega, \hat{a}_{\omega'}^\dagger] = \delta(\omega - \omega')$$

$$\hat{P}^N(x,t) = \int_0^\infty d\omega f(x,\omega)\sqrt{\frac{\hbar}{2\omega}}\left[\hat{a}_\omega e^{-i\omega t} + \hat{a}_\omega^\dagger e^{i\omega t}\right]$$

Response function↔ Coupling function

$$\chi(t-t') = \int_0^\infty d\omega \frac{f^2(\omega)}{\omega} \sin[\omega(t-t')] \mapsto f(\omega) = \sqrt{\frac{2\omega}{\pi} \mathrm{Im}[\chi(\omega)]}$$

$$\ddot{\hat{x}} + \omega_\circ^2 \hat{x} + \partial_t \int_{-\infty}^{t} dt'\, \chi(t-t')\dot{\hat{x}}(t') = -\dot{\hat{P}}^N(t) = \hat{F}^N(t)$$

Example: Set $\chi(t-t') = 2\gamma\theta(t-t')$ then

$$\ddot{\hat{x}} + 2\gamma\dot{\hat{x}} + \omega_\circ^2 \hat{x} = \hat{F}^N(t)$$

$$\hat{P}^{N+}(\omega) = \sqrt{\frac{\hbar\pi}{\omega}} f(\omega)\hat{a}_\omega$$

$$\langle \hat{P}^{N-}(\omega)\hat{P}^{N+}(\omega')\rangle = 2\hbar\, \mathrm{Im}[\chi(\omega)] \frac{1}{e^{\beta\hbar\omega}-1} \delta(\omega-\omega')$$

Hamiltonian: Minimal Coupling Method

$$H = \sum p_i \dot{q}_i - L$$

Minimal coupling K[8]

$$\Downarrow$$

$$H = \frac{(p - P)^2}{2} + \frac{1}{2}\omega_\circ^2 x^2 + \frac{1}{2}\int_0^\infty d\omega\,[P_\omega^2 + \omega^2 Y_\omega^2]$$

$$P = \int_0^\infty d\omega\, f(\omega) Y_\omega$$

$$H_{int} = -pP$$

Fermi's golden rule

$$\Gamma = \frac{2\pi}{\hbar^2} \sum_f |\langle f | H_{int} | 0 \rangle|^2 \delta(\omega - \omega')$$

$$\Downarrow$$

The probability rate for transitions $|n\rangle \to |n \pm 1\rangle$ are given by

$$\Gamma_{|n\rangle \to |n-1\rangle} = \frac{n\omega_\circ \pi}{\hbar} |f(\omega_\circ)|^2 \frac{e^{\beta\hbar\omega_\circ}}{e^{\beta\hbar\omega_\circ} - 1},$$

$$\Gamma_{|n\rangle \to |n+1\rangle} = \frac{n\omega_\circ \pi}{\hbar} |f(\omega_\circ)|^2 \frac{1}{e^{\beta\hbar\omega_\circ} - 1},$$

where $\beta = \frac{1}{k_B T}$. At $T = 0$ there is only dissipation. This formalism can be generalized to amplifying media K[9].

Static magnetodielectric medium

Note that electromagnetic field is a collection of harmonic oscillators and we know how to quantize an oscillator in the presence of its environment so what follows is a straightforward generalization.

Temporal gauge: $A^0 = 0 \Rightarrow \mathbf{E} = -\partial_t \mathbf{A}, \quad \mathbf{B} = \nabla \times \mathbf{A}$

$$
\begin{aligned}
\mathcal{L} \;&=\; \frac{1}{2}\epsilon_0 \, (\partial_t \mathbf{A})^2 - \frac{1}{2\mu_0} (\nabla \times \mathbf{A})^2 \\
&+\; \frac{1}{2}\int_0^\infty d\nu \, [(\partial_t \mathbf{X})^2 - \nu^2 \mathbf{X}^2] \rightarrow \text{elec. properties} \\
&+\; \frac{1}{2}\int_0^\infty d\nu \, [(\partial_t \mathbf{Y})^2 - \nu^2 \mathbf{Y}^2] \rightarrow \text{magn. properties} \\
&-\; \epsilon_0 \int_0^\infty d\nu \, f_{ij}(\mathbf{r}, t, \nu) X^j \partial_t A_i \rightarrow (\mathbf{P} \cdot \mathbf{E}) \\
&+\; \frac{1}{\mu_0}\int_0^\infty d\nu \, g_{ij}(\mathbf{r}, t, \nu) Y^j (\nabla \times \mathbf{A})_i \rightarrow (\mathbf{M} \cdot \mathbf{B})
\end{aligned}
$$

Static magnetodielectric medium

Definition of polarizations:

$$
\begin{aligned}
P_i(\mathbf{r}, t) &= \epsilon_0 \int_0^\infty d\nu\, f_{ij}(\mathbf{r}, t, \nu) X^j, \\
M_i(\mathbf{r}, t) &= \frac{1}{\mu_0} \int_0^\infty d\nu\, g_{ij}(\mathbf{r}, t, \nu) Y^j
\end{aligned}
$$

Following the same steps for harmonic oscillator we have

$$
\mathbf{P}(\mathbf{r}, \omega) = \mathbf{P}^N(\mathbf{r}, \omega) + \epsilon_0^2 \int_0^\infty d\nu\, \frac{f_{ij}\, f_{kj} E_k}{\nu^2 - \omega^2},
$$

$$
\mathbf{M}(\mathbf{r}, \omega) = \mathbf{M}^N(\mathbf{r}, \omega) + \frac{1}{\mu_0^2} \int_0^\infty d\nu\, \frac{g_{ij}\, g_{kj} B_k}{\nu^2 - \omega^2},
$$

Static magnetodielectric medium

Define response tensors by:

$$\chi^e_{ik}(\mathbf{r},\omega) = \epsilon_0 \int_0^\infty d\nu \, \frac{f_{ij}(\mathbf{r},\nu) f_{kj}(\mathbf{r},\nu)}{\nu^2 - \omega^2},$$
$$\chi^m_{ik}(\mathbf{r},\omega) = \frac{1}{\mu_0} \int_0^\infty d\nu \, \frac{g_{ij}(\mathbf{r},\nu) g_{kj}(\mathbf{r},\nu)}{\nu^2 - \omega^2},$$

We can assume $f = f^t$ and $g = g^t$, therefore:

$$\bar{f}(\mathbf{r},\omega) = \sqrt{\frac{2\omega}{\pi\epsilon_0} \mathrm{Im}\bar{\chi}^e(\mathbf{r},\omega)},$$
$$\bar{g}(\mathbf{r},\omega) = \sqrt{\frac{2\omega\mu_0}{\pi} \mathrm{Im}\bar{\chi}^m(\mathbf{r},\omega)},$$

Static magnetodielectric medium

$$\nabla\times(\frac{\mathbf{1}}{\bar{\mu}}\cdot\nabla\times\mathbf{E})-\frac{\omega^2}{c^2}\,\bar{\epsilon}\cdot\mathbf{E}=\mu_0\omega^2\mathbf{P}^N+i\mu_0\omega\nabla\times\mathbf{M}^N$$

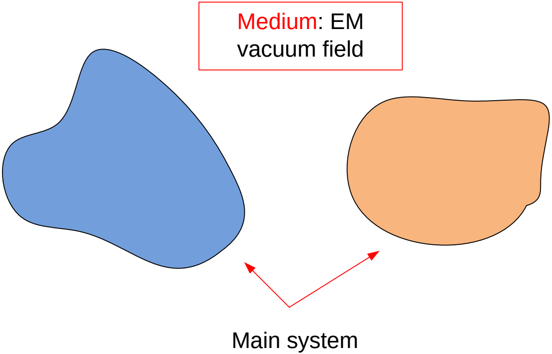
where

$$\begin{aligned}\bar{\mu}(\mathbf{r},\omega) &= \frac{\mathbf{1}}{\mathbf{1}-\bar{\chi}^m(\mathbf{r},\omega)}, \quad \text{magn. permeability}\\ \bar{\epsilon}(\mathbf{r},\omega) &= \mathbf{1}+\bar{\chi}^e(\mathbf{r},\omega), \quad \text{elec. permittivity}\end{aligned}$$

For non magnetic and isotropic matter we have

$$\nabla\times\nabla\times\mathbf{E}-\frac{\omega^2}{c^2}\,\epsilon(\mathbf{r},\omega)\mathbf{E}=\mu_0\omega^2\mathbf{P}^N$$

Example 4: Casimir effect



Casimir effect

Total Lagrangian density:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\epsilon_0(\frac{\partial \mathbf{A}}{\partial t})^2 - \frac{1}{2\mu_0}(\nabla\times\mathbf{A})^2 \\ &+ \frac{1}{2}\int_0^\infty d\nu\,[(\frac{\partial \mathbf{X}_\nu}{\partial t})^2 - \nu^2\mathbf{X}_\nu^2] \\ &- \epsilon_0\int_0^\infty d\nu\,(\frac{\partial \mathbf{A}}{\partial t})\cdot\bar{f}\cdot\mathbf{X}\end{aligned}$$

Wick rotation:

$$it = \tau \;\; \rightarrow dt = -id\tau, \;\; \partial_t = i\partial_\tau$$

$$iS = i\int d\mathbf{r}\int_0^t dt\,\mathcal{L} \rightarrow \int d\mathbf{r}\int_0^\beta d\tau\,\mathcal{L}_E$$

Casimir effect

Euclidean Lagrangian density:

$$\begin{aligned}\mathcal{L}_E &= -\frac{1}{2}\mathbf{A}\cdot\underbrace{\left(-\epsilon_0\frac{\partial^2}{\partial\tau^2}+\frac{1}{\mu_0}\nabla\times\nabla\times\right)}_{\bar{D}}\cdot\mathbf{A}\\ &- \frac{1}{2}\int_0^\infty d\nu\,\mathbf{X}\cdot\underbrace{\left(-\frac{\partial^2}{\partial\tau^2}+\nu^2\right)}_{\bar{B}}\cdot\mathbf{X}\\ &+ \epsilon_0\int_0^\infty d\nu\,\mathbf{A}\cdot\bar{f}\cdot\frac{\partial\mathbf{X}}{\partial t}\end{aligned}$$

$$Z=\int D[\mathbf{A}]\prod_{\nu\geq 0}D[\mathbf{X}_\nu]\,e^{S_E[\mathbf{A},\{\mathbf{X}_\nu\}]}=tr\,e^{S_E[\mathbf{A},\{\mathbf{X}_\nu\}]}$$

Casimir effect

$$Z = \int \prod_{\nu \geq 0} D[\mathbf{X}_\nu] D[\mathbf{A}]\, e^{-\frac{1}{2}\int d\mathbf{r} \int_0^\beta d\tau [\mathbf{A} \cdot \bar{D} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{J}]}$$

$$\times \quad e^{-\frac{1}{2}\int d\mathbf{r} \int_0^\beta d\tau \int_0^\infty d\nu\, \mathbf{X}_\nu \cdot \bar{B} \cdot \mathbf{X}_\nu}$$

where

$$\mathbf{J} = \int_0^\infty d\nu\, \bar{f} \cdot \frac{\partial \mathbf{X}_\nu}{\partial \tau}$$

$$
\begin{aligned}
\mathbf{A}(\mathbf{r},\tau) &= \sum_{n=0}^{\infty}{}' [\mathbf{A}_n(\mathbf{r}) e^{-i\omega_n\tau} + \mathbf{A}_n^*(\mathbf{r}) e^{i\omega_n\tau}] \\
\mathbf{X}_\nu(\mathbf{r},\tau) &= \sum_{n=0}^{\infty}{}' [\mathbf{X}_{\nu,n}(\mathbf{r}) e^{-i\omega_n\tau} + \mathbf{X}_{\nu,n}^*(\mathbf{r}) e^{i\omega_n\tau}]
\end{aligned} \tag{1}
$$

where $\omega_n = \frac{2\pi n}{\beta}$ are Matsubara frequencies for bosonic fields.

$$
\int_0^\beta e^{i(\omega_n - \omega_m)\tau} d\tau = \beta \delta_{nm}
$$

Casimir effect

$$
\begin{aligned}
Z &= \int \prod_{n,\nu\geq 0} D[\mathbf{X}_{\nu,n}]D[\mathbf{X}^*_{\nu,n}] \prod_{n\geq 0} D[\mathbf{A}_n]D[\mathbf{A}^*_n] \\
&\times \; e^{-\frac{1}{2}\int d\mathbf{r} \sum_{n=0}^{\infty}{}' (\mathbf{A}_n\cdot\beta\bar{D}\cdot\mathbf{A}^*_n + \mathbf{A}^*_n\cdot\beta\bar{D}\cdot\mathbf{A}_n + \mathbf{A}_n\cdot\mathbf{J}^*_n + \mathbf{A}^*_n\cdot\mathbf{J}_n)} \\
&\times \; e^{-\frac{1}{2}\int d\mathbf{r} \int_0^\infty d\nu\, (\mathbf{X}^*_{\nu,n}\cdot\beta\bar{B}\cdot\mathbf{X}_{\nu,n} + \mathbf{X}_{\nu,n}\cdot\beta\bar{B}\cdot\mathbf{X}^*_{\nu,n})}
\end{aligned}
$$

Now we integrate over EF degrees of freedom.

Casimir effect

$$
\begin{aligned}
Z &= \underbrace{\prod_{n\geq 0}^{\prime}(\det[\beta\bar{D}])^{-1}}_{\text{partition function of free EF}} \int \prod_{n,\nu\geq 0} D[\mathbf{X}_{\nu,n}]D[\mathbf{X}^*_{\nu,n}] \\
&\times \; e^{-\frac{1}{2}\int d\mathbf{r}\int_0^\infty d\nu\,(\mathbf{X}^*_{\nu,n}\cdot\beta\bar{B}\cdot\mathbf{X}_{\nu,n}+\mathbf{X}_{\nu,n}\cdot\beta\bar{B}\cdot\mathbf{X}^*_{\nu,n})} \\
&\times \; e^{\int\int d\mathbf{r}d\mathbf{r}'\,\mathbf{J}^*_n(\mathbf{r})\cdot\frac{1}{\beta}\bar{G}\cdot\mathbf{J}_n(\mathbf{r}')}
\end{aligned}
$$

where $\bar{G}_0 = \bar{D}^{-1}$ is the free EF dyadic Green's function $\bar{D}\bar{G}_0 = \mathbb{I}$.

Casimir effect

Now we integrate over matter degrees of freedom in a similar way to find

$$
\begin{aligned}
Z &= \underbrace{\prod_{n\geq 0}{}' (\det[\beta \bar{D}])^{-1}}_{Z^0_{EF}} \underbrace{\prod_{n\geq 0}{}' (\det[\beta \bar{B}])^{-1}}_{Z^0_{M}} \\
&\times \underbrace{\prod_{n\geq 0}{}' (\det[1 + \omega_n^2 G_B \bar{f}^t \cdot \bar{G}_0 \cdot \bar{f}])^{-1}}_{\color{red}{Z_{eff}}}
\end{aligned}
$$

Now using $\ln[\det \hat{O}] = tr \ln[\hat{O}]$ we find

Casimir effect

$$\ln Z_{eff} = -\sum_{n=0}^{\infty}{}' \, tr \ln[1 + \omega_n^2 G_B \cdot \bar{f}^t \cdot \bar{G}_0 \cdot \bar{f}]$$

$$\ln(1+x) = \sum_{m=1}^{\infty} (-1)^{m-1} \frac{x^m}{m}$$

$$\chi_{ik}(\mathbf{r}, \omega) = \int_0^{\infty} d\nu \, \frac{f_{ij}(\mathbf{r}, \nu) f_{kj}(\mathbf{r}, \nu)}{\nu^2 - \omega^2}$$

$$\bar{\chi}(\mathbf{r}, i\omega_n) = tr_\nu[G_B(i\omega_n)\bar{f}(\mathbf{r})\bar{f}^t(\mathbf{r})]$$

Casimir effect

$$\ln Z_{eff} = -\sum_{n=0}^{\infty}{}' tr_{|i,\mathbf{r}\rangle} \ln[\underbrace{1 + \bar{\chi}(i\omega_n) \cdot \bar{G}_0(i\omega_n)}_{\bar{G}\cdot\bar{G}_0^{-1}}]$$

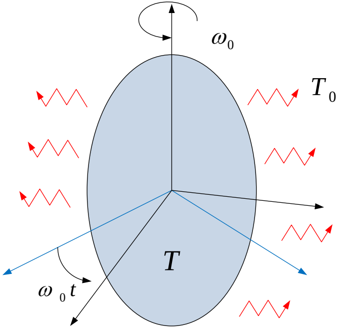
The free energy is defined by

$$F = -k_B T \ln Z_{eff} = k_B T \sum_{n=0}^{\infty}{}' tr_{|i,\mathbf{r}\rangle} \ln[1 + \bar{\chi}(i\omega_n) \cdot \bar{G}_0(i\omega_n)]$$

In zero temperature $\int_0^\infty \frac{d\zeta}{2\pi} \leftrightarrow k_B T \sum_{n=0}^{\infty}{}'$

$$F = \int_0^\infty \frac{d\zeta}{2\pi} \, tr \ln[1 + \bar{\chi}(i\zeta) \cdot \bar{G}_0(i\zeta)]$$

Rotating Dielectric



$$\rho' = \rho, \;\; \varphi' = \varphi - \omega_0 t, \;\; z' = z, \;\; t' = t,$$
$$\partial_{\rho'} = \partial_\rho, \;\; \partial_{\varphi'} = \partial_\varphi, \;\; \partial_{z'} = \partial_z, \;\; \partial_{t'} = \partial_t + \omega_0 \partial_\varphi,$$

Rotating Dielectric

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\epsilon_0 \left(\partial_t \mathbf{A}\right)^2 - \frac{1}{2\mu_0}(\nabla\times\mathbf{A})^2 \\ &+ \frac{1}{2}\int_0^\infty d\nu \left[(\partial_t \mathbf{X} + \omega_0 \partial_\varphi \mathbf{X})^2 - \nu^2 \mathbf{X}^2\right] \\ &- \epsilon_0 \int_0^\infty d\nu\, f_{ij}(\nu, t) X^j \partial_t A_i \\ &+ \epsilon_0 \int_0^\infty d\nu\, f_{ij}(\nu, t) X^j (\mathbf{v} \times \nabla\times\mathbf{A})_i \end{aligned}$$

Coupling tensor

Coupling tensor is now time-dependent

$$f_{ij}(\nu, t) = \begin{pmatrix} f_{xx}(\nu)\cos(\omega_0 t) & f_{xx}(\nu)\sin(\omega_0 t) & 0 \\ -f_{yy}(\nu)\sin(\omega_0 t) & f_{yy}(\nu)\cos(\omega_0 t) & 0 \\ 0 & 0 & f_{zz}(\nu) \end{pmatrix}$$

We assume $f_{xx} = f_{yy}$ in body frame.

Lagrangian can be generalized to a covariant one including the magnetic properties.

Main equation

Main equation

$$[\nabla\times\nabla\times \; - \frac{\omega^2}{c^2}\mathbb{I} - \frac{\omega^2}{c^2}\tilde{\mathbb{D}}\cdot\boldsymbol{\chi}^{ee}(\omega, -i\partial_\varphi)\cdot\mathbb{D}]\cdot\mathbf{E} = \mu_0\omega^2\tilde{\mathbb{D}}\mathbf{P}^N$$

where $\mathbb{D} = \mathbf{1} + \frac{1}{i\omega}\mathbf{v}\times\nabla\times$ and $\tilde{\mathbb{D}} = \mathbf{1} + \frac{1}{i\omega}\nabla\times\mathbf{v}\times$. The presence of operators $\mathbb{D}, \tilde{\mathbb{D}}$ in this recent equation makes it a complicated equation. For small velocity regime ($v/c \ll 1$) we can set approximately $\mathbb{D}, \tilde{\mathbb{D}} \approx 1$ and in high velocity regime numerical calculations may be applied.

Fluctuation-Dissipation relations

$$\langle P_i^N(\mathbf{r},\omega)P_j^{N\dagger}(\mathbf{r}',\omega')\rangle = 4\pi\epsilon_0\hbar\,\delta_{ij}\Gamma_{ij}(\omega,-i\partial_\varphi)\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega')$$

where Γ_{ij} are defined by $\Gamma_{xz}=\Gamma_{zx}=\Gamma_{yz}=\Gamma_{zy}=0$,

$$\Gamma_{zz}(\omega,m) = 2\mathrm{Im}[\chi^0_{zz}(m\omega_0-\omega)]a_T(m\omega_0-\omega),$$

$$\begin{aligned}\Gamma_{xx}(\omega,m) &= \mathrm{Im}[\chi^0_{xx}(m\omega_0-\omega_+]a_T(m\omega_0-\omega_+)\\ &+ \mathrm{Im}[\chi^0_{xx}(m\omega_0-\omega_-]a_T(m\omega_0-\omega_-),\end{aligned}$$

$$\begin{aligned}\Gamma_{xy}(\omega,m) &= i\,\mathrm{Im}[\chi^0_{xx}(m\omega_0-\omega_-]a_T(m\omega_0-\omega_-)\\ &- i\,\mathrm{Im}[\chi^0_{xx}(m\omega_0-\omega_+]a_T(m\omega_0-\omega_+),\end{aligned}$$

and $a_T(\omega)=\coth(\hbar\omega/2k_BT)=2[n_T(\omega)+\frac{1}{2}]$

Hamiltonian

$$
\begin{aligned}
H &= \int_V d\mathbf{r}\Big\{\frac{1}{2\epsilon_0}(\mathbf{P}-\mathbf{D})^2+\frac{1}{2\mu_0}(\nabla\times\mathbf{A})^2 \\
&+ \frac{1}{2}\int_0^\infty d\nu\,[\mathbf{Q}_\nu^2+\nu^2\mathbf{X}_\nu^2] \\
&- \omega_0\int_0^\infty d\nu\,\mathbf{Q}_\nu\cdot\partial_\varphi\mathbf{X}_\nu-\mathbf{P}\cdot(\mathbf{v}\times\nabla\times\mathbf{A})\Big\}
\end{aligned}
\tag{2}
$$

Interaction:

$$
\begin{aligned}
H_{int} &= -\int_{V_s} d\mathbf{r}[\mathbf{P}(\mathbf{r},t)\cdot\mathbf{E}(\mathbf{r},t) \\
&+ \mathbf{P}(\mathbf{r},t)\cdot(\mathbf{v}\times\nabla\times\mathbf{A}(\mathbf{r},t))], \\
&= -\int_V d\mathbf{r}[\mathbf{P}(\mathbf{r},t)\cdot\mathbf{E}(\mathbf{r},t)+(\mathbf{P}(\mathbf{r},t)\times\mathbf{v})\cdot\mathbf{B}(\mathbf{r},t)]
\end{aligned}
\tag{3}
$$

The radiated power

The radiated power can be written as

$$\langle \mathcal{P} \rangle = - \int_V d\mathbf{r} \langle [\partial_t \mathbf{P} - \nabla \times (\mathbf{v} \times \mathbf{P})] \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \rangle$$

where $| \rangle = | vacuum \rangle_{T_0} \otimes | matter \rangle_T$ is the tensor product of initial thermal states of the electromagnetic and matter field which are supposed to be held at temperatures T_0 and T respectively. For small bodies or small velocities we have

$$\langle \mathcal{P} \rangle = - \int_{V_s} d\mathbf{r} \langle \partial_t \mathbf{P} \cdot \mathbf{E} \rangle. \tag{4}$$

The radiated power

For an extended body with azimuthal symmetry and small velocity we have

$$\langle \mathcal{P} \rangle = \frac{\hbar}{2\pi c^2} \int d\mathbf{r} \int_{-\infty}^{\infty} d\omega \omega^3 [a_T(\omega - \omega_\circ \hat{l}_z) - a_{T_\circ}(\omega)]$$

$$\{ \mathrm{Im}\chi^\circ_{zz}(\omega - \omega_\circ \hat{l}_z) \mathrm{Im}\, G_{zz}(\mathbf{r}, \mathbf{r}', \omega) + \mathrm{Im}\chi^\circ_{xx}(\omega - \omega_\circ \hat{l}_z)$$

$$\times \mathrm{Im}[G_{xx}(\mathbf{r}, \mathbf{r}', \omega) + G_{yy}(\mathbf{r}, \mathbf{r}', \omega)] \cos(\varphi - \varphi')| \}_{\mathbf{r}' \to \mathbf{r}}$$

where $\hat{l}_z = -i\partial_\varphi$, and we used the symmetry properties of tensors $G_{ij}(\mathbf{r}, \mathbf{r}', \omega)$ and $\Gamma_{ij}(\omega, -i\partial_\varphi)$.

Spherical Drude particle: PRL 105, 113601 (2010)

$$\omega(t) = \omega_0 e^{-\frac{t}{\tau}}$$

$$\tau(\text{Stopping time}) = \frac{(\hbar c)^3}{\pi} \frac{\rho a^2 \sigma_\circ}{(k_b T_\circ)^4}$$

$\rho =$ Particle density
Graphite particles are abundant in interstellar dust [F. Hoyle and N. C. Wickramasinghe, Mon. Not. R. Astron. Soc. 124, 417 (1962)
$\sigma_0 = 2.3 \times 10^4 (2.0 \times 10^5), \quad a = 10(100)nm,$
For $a = 10nm, \ T_0 = 1000K \rightarrow \tau \approx 1$Day
For $a = 10nm, \ T_0 \approx$ room temperature $\rightarrow \tau \approx 1$Year
For $a = 100nm, \ T_0 = 2.7K \ \rightarrow \tau \sim 0.6$ billion years

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